The Efficiency-Stability Tradeoff: How efficient world order drives catastrophic war contagion.

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Abstract

Why do states construct fragile peace settlements that risk catastrophic war contagion? We study a multi-actor, multi-issue model of world order. We examine how variation in the amount that states extract from a settlement affects the scope of war contagion should world order collapse. We show that states face an efficiency-stability trade-off. Settlements that leave states with their favorite issues risk war contagion if peace collapses. Conversely, settlements that split issues proportionate to power are stable but inefficient. We use the trade-off to illuminate important differences between bipolar and multipolar systems. In multipolar systems, agreements may violate the bounds of plausible settlements defined in Fearon (1995) or involve a nonzero risk of war. We contribute to research on the causes of war, war contagion, international order, and diplomacy.

Leading historians describe Europe circa 1900 as a 'doomsday machine.' The European powers had constructed a peace so complicated that a small accident anywhere could cause war everywhere. Do certain kinds of peace settlements drive war contagion? If they do, who would construct a peace so fragile that contagion was inevitable? We analyze a negotiation model of war with multiple actors who contest multiple issues.[1] Each issue represents a distinct theatre of conflict, and settlements represent world order forged between the great powers either in peacetime or in a postwar moment (Ikenberry 2001). We argue that states often desire a world order that risks catastrophic war contagion if peace collapses (Levy 1982; Vasquez 2018). We explain the scale of wars (Weisiger 2013; Gartner and Siverson 1996) and the number of belligerents dragged into conflict (Christensen and Snyder 1990; Trager 2015) as the result of seemingly well-functioning world order in the years beforehand (Braumoeller 2013). We also provide one explanation for why rational, fully-informed states who face no commitment problems initiate war in a multi-polar system, but never in a bipolar system (Waltz 1979).

Our theory relies on an efficiency-stability trade-off. Efficient settlements are possible because states contest multiple issues and value some issues more than others (Joseph 2020). States trade the issues they care about least in return for issues they care about most (Trager 2011). However, a sinister promise underpins efficient settlements. The simplest version of this promise is between two states: since I kept all of one issue, you may be tempted to fight me for it; if you do, I will expand the war to the issue you kept. States do not initiate war because they anticipate reciprocal war expansion. But if a hazard (such as an accident, uncertainty, third-party intervention, or a commitment problem) threatens to plunge states into a local conflict, the war expands. This settlement is efficient but risks contagion—the spread of war from one issue to multiple issues. Alternatively, states might negotiate a stable settlement, wherein they split each issue proportionate to their relative power. This settlement is inefficient, because states do not get the most of their favorite issues, but it carries no risk of contagion if peace breaks down.

We use the trade-off to illuminate two important findings for the scope and timing of war. First, we introduce re-negotiation (Wagner 2000) and the risk of an accident (Schelling 1957). This allows us to ask: when an unanticipated crisis would trigger contagion, why do states fail to re-negotiate an inefficient but non-contagious peace? We show that if the efficient settlement favors one state more than the other, or if one state is power preponderant, then states have divergent preferences regarding efficiency versus stability.

[1] Formally, we model states that inherit a settlement. Their strategies are conditional war declarations. We characterize all the settlements that could sustain peace for a specific punishment strategy. This allows us to connect the nature of peace settlements to the risk of war contagion.

One state calls for renegotiation because he prefers an efficient deal with no contagion. The other refuses to renegotiate because she prefers efficient peace that risks contagion.

Second, we use the efficiency-stability trade-off to offer a rationalist explanation for war initiation in a multipolar world. Scholars have long wondered why states would initiate crises that risk unravelling complex international systems. They explain war initiation through miscalculation, private information, or domestic incentives (see Ramsay 2017; Jervis 1985; Wagner 1994; Waltz 1979; Smith 2021, for discussion). We show that clever diplomats may deliberately construct a world order that introduces a nonzero probability of war initiation to extract an unusually profitable bargain while peace prevails.

The logic of war initiation is as follows. Consider a world with three great powers—A, B, and C—in which A has proposal power. A can construct a peace settlement that is so profitable to her that she can only threaten to retaliate when she faces challenges from both B and C. If instead, only one state challenges A, war is not contagious. B and C face competing incentives to initiate bilateral wars against A. Each knows if he starts a local war alone, he will do better than accepting peace. But if both start wars, then they trigger a global conflict that leaves everyone worse off. In equilibrium, B and C are both dissatisfied with peace, but they only probabilistically initiate local conflicts. A cannot orchestrate this deal in a bipolar world because she needs two rivals to play against each other.

We illustrate our theory using two understudied treaties and their aftermaths: the Peace of Utrecht following the War of the Spanish Succession (1701-1714) and the Treaty of Aix-la-Chapelle following the War of the Austrian Succession (1740-1748).[2] These two treaties differed in how efficiently they distributed territory in peace. The Peace of Utrecht was inefficient because states agreed to borders that largely matched the military situation at the end of the war, even though several actors would have preferred to negotiate a more advantageous exchange. In contrast, the Treaty of Aix-la-Chapelle was more efficient because states exchanged territory to match their preferences (e.g. Great Britain yielded conquests in North America and France in the Low Countries). Under these different world orders, patterns of war contagion were likewise different. Under the inefficient Utrecht settlement, several bilateral crises broke out, but the subsequent conflicts involved few belligerents and issues. However, the first bilateral crisis under the efficient Treaty of Aix-la-Chapelle quickly escalated into the Seven Years' War (1756-1763), which dragged five great powers into conflict across multiple continents.

[2]A handful of scholars have used these cases to illuminate social science theories, e.g. Mitzen (2013) uses Aix-la-Chapelle, Levy (1981) uses Utrecht, and Braumoeller (2013); Cesa (2010) use both.

Most broadly, we contribute to the literature on the structural causes (Braumoeller 2008) and scope (Weisiger 2013; Debs 2020) of war between major powers (Vasquez 1993; Levy and Vasquez 2014). We contribute most directly to war contagion theory (Vasquez 2018). We accept that tangible connections between states—trade interdependencies (Barbieri and Levy 1999), alliances (Smith 2021), geography (Mearsheimer 2001), hierarchy affiliation (Braumoeller 2019), and technology transfers (Bas and Coe 2012)—amplify the risk of war contagion; and that these connections match the pathways along which war contagion occurs in real life. We show that who extracts what from whom in the initial construction of peace likely contributes to the interdependencies that form. Thus, we expose the bargaining pre-conditions that shape alliances and other dependencies that drive patterns of war contagion in any specific case. We also contribute directly to the study of diplomacy by providing one of the first formal analyses of negotiations during peacetime diplomatic conferences designed to forge world order before a crisis erupts. These peace-time diplomatic actions are distinct from crisis negotiations (e.g. Trager 2016).[3] Scholars have shown that crisis negotiations can prevent wars from starting (Fearon 1995). We argue that conference negotiations affect both the incentives to start wars and the scope of war if a future crisis is not peacefully resolved.

We highlight the dark-side of issue linkage (Poast 2012; Keohane 2005). Issue linkage creates efficiencies if peace holds. But it requires that any bilateral crisis sparks contagious conflict. This highlights the grave consequences American policymakers could face if they complicate the existing world order to sustain peace with China, Russia, or others (Simmons and Goemans 2021; Adler-Nissen and Zarakol 2021). The price of more complex agreements may be a larger risk of catastrophic war should peace collapse.

Our formal model connects two strands of research on bargaining and war: multi-actor models and multi-issue models. Like multi-actor models (Wagner 2007; Wolford 2020; Niou, Ordeshook, and Rose 1989; Gartner and Siverson 1996; Benson and Smith 2022) we allow states to simultaneously decide to initiate war over individual issues *and* to expand the war to new issues after another player initiates. Like multi-issue models (Trager 2011; Joseph 2020) we assume that states contest multiple issues and value some issues more than others. These agendas interact in interesting ways. Heterogeneous preferences are necessary for efficient exchange.But the settlements we can support and the risk of war vary dramatically once we move from two to many actors.

We derive three other notable results. First, we identify an unappreciated *minimum* interest condition

[3] Coalition formation in economics is close (Ray and Vohra 2015). Our model allows for spatial bargaining and multi-issue preferences.

for issue linkage (Maggi 1999; Kennard, Krainin, and Ramsay nd). Second, we show fair division is often rationally-optimal because actors with complete proposal power will nonetheless offer more than a rival's minimum demand from fighting (Massoud 2000). Third, we show that the commonly understood bounds on the distribution of territory that power and the cost of war impose in a dyad (Fearon 1995) do not apply in a multipolar world.

1 Negotiation as Carving World Order

Most bargaining models conceive of negotiation during a specific crisis. They then ask, why did states go to war rather than bargain? However, many important diplomatic negotiations occur in the wake of major wars or when no specific militarized crisis is imminent (Ikenberry 2001). Great Powers use these negotiations to establish the fundamental terms of international peace. We use the term "conference settlement" to mean the shared understanding great powers forge over who gets what, and what happens if one state violates their shared understanding. Depending on the specific case, the contours of world peace may be negotiated at one Conference or at repeated meetings over several years. In some cases, all of the terms are written down in a single document; in others, they are written in several documents or not written at all. Either way, the settlement is the common understanding about how to divide the world and sustain peace in the years that follow. Here are three examples of what we mean.

The Peace of Westphalia Following the Thirty Years' War (1618-1648), the European powers gathered in Westphalia. The treaties of Münster and Onsabrück settled the religious question that had fueled the conflict, changed the constitution of the Holy Roman Empire, and created a new territorial settlement (Wilson 2009, 751-778). The agreement secured general peace in Germany until the War of the Spanish Succession.

The Congress of Vienna Five European Powers redistributed the many territorial conquests of Napoleon following his defeat in 1814. Following the congress, Europe enjoyed an unprecedented 38 years of peace until the Crimean War (1853-1856), followed by the wars of Italian and Germany unification. The agreement's contours survived these conflicts. It finally collapsed with the outbreak of World War I.

Cold War spheres of influence As the Second World War drew to a close, the Allies met at Yalta and Potsdam to discuss the postwar order. Their Foreign Ministers continued to meet throughout the late 1940s to refine and implement these agreements, and these negotiations shifted as Stalin's interests and intentions became apparent. They culminated in an understanding of spheres of influence that served as the basis for peace during the Cold War. The territorial exchanges defined during this period grounded peace until re-negotiated during détente.

These settlements share several features that we embed in our model's assumptions. First, the settlements involved multiple issues, multiple great powers, or both. Second, they were not negotiated during a pre-war, bilateral crisis. Rather, they were negotiated after the end of a major war (e.g. Westphalia) or during peacetime (e.g. détente). Third, the agreements were designed to sustain peace for decades. Fourth, the agreements were sticky: once all of the great powers agreed, it was rare for any bilateral pair to renegotiate their portion of the agreement. In a few cases states marginally adjusted their bilateral relationships between major conferences. But in many cases the Great Powers did not alter the settlement even after an unanticipated power shift or a local crisis. In fact, wars often started out of bilateral crises, leading to war contagion, without states trying to re-negotiate at the global level. For example, the July Crisis between Austria and Serbia led to World War I rather than a re-negotiation between France, Austria, Russia, and Germany.[4]

To be clear, great power satisfaction varies in the years following a conference settlement. In some cases, all great powers believe they got a fair deal. In others, some openly complain about the agreement. We say that peace is sustained so long as no actor revises the agreements through war.

Many historians believe that how much states extract largely depends on the quality of their diplomatic representation. Drawing from these insights, recent social science scholarship has studied proposal power in the shadow of war (see Brutger 2021; Trager 2016). For example, Metternich was a skilled diplomat with enormous proposal power. At Vienna, he crafted a settlement from which Austria extracted a large part of the surplus despite its comparatively weak position (Jarrett 2013). In contrast, in the mid-1940s, the United States and the Soviet Union shared proposal power more-or-less equally. Thus, the resulting peace was more equitable, even though Soviet-American relations were hostile. To explore the connection between proposal power and the scope of war contagion, we analyze different settlements designed to maximize the welfare of different actors.

Besides the distribution of disputed issues, conference settlements also include a tacit or explicit un-

[4]Our baseline theory does not cover crisis negotiations such as Anglo-German negotiations, or Soviet-American negotiations during the Berlin Crisis. We discuss crisis bargaining in Appendix B.1.

derstanding about what happens if peace fails. Thus, a conference settlement is best understood as an equilibrium that sustains the peaceful distribution of territory through the threat of war (Wagner 1994, 2000, 2007). As Slantchev (2005) notes, once the conference concludes states inaugurate a period of peace that we can observe. But equally important is what we *cannot* observe. The settlement is sustained by each state's latent promise (tacit or explicit) of what will happen if another state violates the agreement. Slantchev shows that the Great Powers at Vienna made these threats of war contagion explicit to strengthen the likelihood that peace would succeed.[5] We expand Slantchev's work to connect the nature of peace to the scope of war. Our game theoretic analysis allows us to study the threats necessary to sustain peace. We show that in multipolar systems the same settlement can be sustained by a variety of different background threats, some involving minimal contagion, and some risking world war.

1.1 Road map and summary of findings

Putting it all together, our model starts with great powers in a peaceful (i.e. no war/crisis) world.[6] We assume that peace follows because great powers reached a settlement years ago. Since states start in a period of peace, we do not allow them to make proposals to alter peace in the baseline model. Rather, we examine a world in which there is a status quo settlement. We then allow states to continue living under the terms of peace, or to violate those terms by initiating any local war(s) they choose.

We treat the distribution of issues[7] that follow from a conference settlement as our independent variable. We focus on the difference between efficient and inefficient settlements. Efficiency refers to Pareto-efficiency: no state can alter the terms of world order to improve its welfare without harming another state. We also distinguish between qualitative features of efficient settlements: extractiveness and welfare. Extractivness takes a specific state's perspective and asks how much does that state extract for herself from this settlement? Welfare loosely refers to the "total" benefits that world order provides to the member states. We treat the character of war should peace fail as our dependent variable. We focus on contagious versus non-contagious (or stable) wars. Contagion refers to the risk that a local war will spread to additional actors or issues.[8]

We proceed as follows. First, we study a bipolar (two-state) world to demonstrate the efficiency-stability

[5]Mitzen (2006) argues that protocols help support peace. We view this as distinct but complementary.

[6]We follow Levy's definition that great powers are states that play "a major role in international politics with respect to security-related issues" and at minimum "has relative self-sufficiency with respect to military security" (see Levy 1983, 10-19).

[7]this includes how states agree to divide territory, norms and values, returns from commercial treaties, etc

[8]Stability means war will not spread once it starts.

trade-off in its simplest setting. Connecting our independent and dependent variables, we show that there is an inescapable trade-off between efficiency and contagion.

Second, we introduce re-negotiation and the risk of an accident. We show that if the value from efficient peace is sufficiently high, states prefer to risk a highly contagious war than re-negotiate an inefficient but stable peace. Disputes over re-negotiation emerge between states (one state wants to renegotiate but the other refuses) if one state is power preponderant or the efficient settlement is highly extractive.

Third, we study a multipolar system. In our model, poles are the number of states that can independently initiate or expand bilateral conflicts against other states in the system. They are also part of world order in that they contest issues with other states and agree to distribute them as part of world order.[9] We generate two findings. First, we contrast bipolar and multipolar worlds to refine and clarify Waltz' discussion of multipolar complexity. Specifically, multipolar worlds exploit complicated and interconnected threats of war to support highly extractive peace settlements that cannot be supported in bipolar worlds. Second, we show that peace settlements exist in which a rational, fully informed state probabilistically initiates a local war, knowing there is a risk that their choice will cause war to spread throughout the system. We use this to provide a rationalist explanation for war initiation in multi-polar worlds.

Fourth, we explore the model's implications for the literature on war contagion and diplomacy. Finally, we illustrate the core efficiency-stability trade-off in two critical but understudied cases of European diplomatic history.

2 Bipolar model: Two players, two issues

Two players $j \in A, B$ contest two issues $i \in 1, 2$. The game starts in a period of peace after the great powers have negotiated a settlement. Players then decide simultaneously whether to initiate war over one, both, or neither of these issues. If a war begins over one issue, an attacked state is given the opportunity to expand the war to the other issue.

Define a pre-negotiated settlement as a vector $q : q_i$ with two elements that represent the division of issues 1 and 2. Define a vector ω as a state variable that lists the issues over which players are fighting wars. The game starts in a state of peace ($\omega = \emptyset$), with a pre-negotiated settlement q that is known to both players. Then, the game unfolds as follows:

[9] Levy (1983) notes that polarity is not well defined or used consistently. Our definition is a formalized version that is consistent with the definition he proffers.

1. Players simultaneously decide to accept peace over all issues or declare war over one or both issues they contest. If all states accept peace, the game ends in peace. Otherwise,

2. All players that have had war declared against them simultaneously decide to accept peace over any remaining peaceful issues or to declare war over them.[10]

A strategy for player j is a war rule $w^j|\omega q$ that determines whether state j declares war over issue i given what offers are on the table and what wars have already been declared. Our model differs from other multi-issue models (Joseph 2020; Trager 2011) because we allow states to declare wars one-at-a-time. This allows us to study variation in the pattern and scope of conflict.

Turning to payoffs, we assume that players have heterogeneous preferences. A values the issues α_1 and α_2; B values them β_1, β_2. If the game ends and an issue remains at peace, states split it according to the proposed settlement. If peace prevails over issue i, A gets $\alpha_i q_i$ and B gets $(1 - q_i)\beta_i$. War over issue i causes a costly lottery that A wins with probability $p_i \in (0, 1)$. Both players pay a cost w for each war they fight (If they fight two wars, they pay $2w$). If war prevails over issue i, then A gets $p_i\alpha_i - w$ and B gets $(1 - p_i)\beta_i - w$. For example, if the game ends in a war over issue 1 and peace over issue 2 then A gets $U^A(\omega = 1) = p_1\alpha_1 - w + \alpha_2 q_2$ and B gets $U^B(\omega = 1) = (1 - p_1)\beta_1 - w + (1 - q_2)\beta_2$. If the game ends in total war, then A gets $U^A(\omega = 1, 2) = p_1\alpha_1 - w + p_2\alpha_2 - w$ and B gets $U^B(\omega = 1, 2) = (1 - p_1)\beta_1 - w + (1 - p_2)\beta_2 - w$.

For simplicity, we assume the gains of trade are considerable:

$$\mathcal{A}_1 : \alpha_1 > \beta_1, \alpha_2 < \beta_2$$

Substantively, $\mathcal{A}_1$ represents a setting where states desire different spheres of influence, or have historical and cultural attachments to different parts of the world.[11]

The following condition guarantees that every issue is worth disputing for every player, e.g. A would never accept $q_2 = 0$ unless it enjoyed an appropriately larger share of q_1.

$$\mathcal{C}_1 : w < \min(\beta_1(1 - p_1), \alpha_2 p_2)$$

[10] If we allow any player to expand the war at any moment, we can support grim-trigger sub-games. In the multi-player model below, this dramatically expands the settlements we can support. All the equilibria we describe would remain equilibria if we relaxed this restraint.

[11] We can generate issue linkage in other ranges using comparative advantage across different issues. However, there is a limit.

Proposition 2.6 will address the consequences of relaxing this condition.

Throughout the paper, we analyze subgame perfect equilibria.

2.1 Analysis

In this section, we focus on settlements that sustain peace. That is, we only examine offers in which no state can profitably deviate to war given the conjectured threats. This matches our substantive focus because we are interested in agreements that have been in place for several years.[12]

Our central claim is that there exists an efficiency-stability tradeoff. To appreciate what that means, we need to characterize stability. We say an equilibrium is **stable** if, for any history of the game, no actor initiates or expands a war. Thus, while peace characterizes whether or not states initiate war, stability has the additional restriction that states would not expand a war, were one to begin. In contrast, an equilibrium is **contagious** if an actor might spread war to another issue, should a war ever begin. Both terms describe off-the-path behaviors as well as observed outcomes. Our first task is to show that stable equilibria exist. Proofs are in Appendix A.

Lemma 2.1 ***Stable settlements:*** *A stable settlement always exists in which no player declares war in any sub-game. In any stable settlement* $q^\dagger$*,* $q_1^\dagger \in [p_1 - \frac{w}{\alpha_1}, p_1 + \frac{w}{\beta_1}]$ *and* $q_2^\dagger \in [p_2 - \frac{w}{\alpha_2}, p_2 + \frac{w}{\beta_2}]$*.*

Stable settlements require two self-contained bargains, where each pie is divided proportionate to each player's power (approximately). Thus, consistent with Fearon (1995)'s 'bargaining range' logic, no matter what else happens in other theaters, players prefer peace to war in each self-contained region.

We are now ready to characterize our main result:

Proposition 2.2 *There is an* ***efficiency-stability trade-off****. Under* $\mathcal{C}_1$*, every stable equilibrium is Pareto-dominated by a contagious equilibrium in which a local war must expand once war starts.*

The offers that support stable equilibria must lie within Fearon's bargaining range. This requires states to divide both issues roughly proportionate to power. But both states have a favourite issue. They can do better if each takes more of the issue she values the most in exchange for the issue she values the least.

The trouble with efficient exchange is that each actor extracts so much of her favourite issue that the other becomes willing to fight a local war over it. Thus, states face a problem: they want efficient exchange,

[12]There exists an equilibrium where both states declare war instantly. We ignore it because it does not fit our substantive focus, and it is Pareto-dominated by any equilibrium that sustains peace.

but they cannot commit not to fight over their least favourite issue. They resolve this problem by linking their offers through the threat of reciprocal war contagion. Thus, each state knows that if she starts a local war, the war will inevitably expand, and she will wish that the war had never started.

What do these contagious and more efficient equilibria look like? In Appendix A.6 we show that we can write out all the Pareto-dominant equilibria as a continuous, linear function of q_1, q_2. Below we examine the two cases that sit at the extreme ends of this linear function. These two cases map onto two common objectives in international relations. The first fits an intuitive understanding of "welfare maximizing" settlements. These settlements are designed to generate the largest possible amount of global utility.[13] The second are extractive settlements, in which one player extracts as much as possible while peace is sustained.[14] When we expand the model to study multipolar worlds, we will push the logic of efficient extraction further. Thus, appreciating the bipolar case will help us with future comparisons.

Lemma 2.3 *In a "welfare-maximizing" equilibrium, each player enjoys all of its favorite issue peacefully, i.e.* $q_1 = 1$ *and* $q_2 = 0$*. A welfare-maximizing equilibrium exists iff* $2w > \max(\alpha_2 p_2 - \alpha_1(1-p_1), \beta_1(1-p_1) - \beta_2 p_2)$*. This settlement can only be peacefully sustained with a reciprocal threat of war expansion.*

Lemma 2.4 *A "maximally-extractive" equilibrium maximizes A's utility. In this equilibrium,* $q_1 = 1$ *and* $q_2 = \min(p_2 - \frac{w}{\alpha_2}, \frac{2w - \beta_1(1-p_1) + \beta_2 p_2}{\beta_2})$*. This settlement can only be peacefully sustained with a reciprocal threat of war expansion.*

How much more efficient are these Pareto-efficient offers than non-contagious offers? Table 1 explores this question with an example that holds the underlying modeling parameters constant. Section 1 explores the most efficient stable equilibria. Section 2 illustrates the most efficient equilibria (which happen to be supported by the reciprocal threat of war expansion). These results illustrate that efficient but unstable equilibria can take many forms. However, they are united by the fact that players extract more utility from them then they would have in stable equilibria.

[13] The concept of "welfare-maximizing" is consistent with historical situations in which states hope to construct an international system in which all states are allowed to profit, e.g. the Liberal International Order (Ikenberry 1998). It similarly aligns with the Rawlsian imperative to maximize the welfare of the least well-off.

[14] This is the result we would obtain if we assumed a state made a take-it-or-leave-it offer.

Table 1: Settlements that can sustain peace for contagious and stable threats of war

		Settlement:	A's utility	B's utility
Sec 1: The stable equilibria that:				
(a)	Maximize A's utility	$q_1 = p_1 + w/\beta_1, q_2 = p_2 + w/\beta_2$	1.177	1*
(b)	Maximize "total welfare"	$q_1 = p_1 + w/\beta_1, q_2 = p_2 - w/\alpha_2$	1.05	1.46
Sec 2: The contagious equilibria that:				
(a)	Maximize A's utility	$q_1 = 1, q_2 = p_2 - \frac{w}{\alpha_2}$	1.65	1.26**
(b)	Maximize "total welfare"	$q_1 = 1, q_2 = 0$	1.5	1.8
Sec 3: Total war:				
	War payoff	$U^A = \alpha_1 p_1 + \alpha_2 p_2 - 2w$ $U^B = \beta_1(1-p_1) + \beta_2(1-p_2) - 2w$	0.65	1

Assumes $\alpha_1 = 1.5, \alpha_2 = \beta_1 = 0.5, \beta_2 = 1.8, w = .1, p_1 = 0.4, p_2 = 0.5$.
Illustrating proposition 2.6, $*$ leaves B with his minimum demand from fighting, but $**$ leaves B with more.

2.1.1 Issue linkage through connected threats of war

We find that even under anarchy states can connect threats of war to support efficient exchange. This resembles classic arguments about institutional issue-linkage (Keohane 2005; Poast 2012). However, our results have two unique features that we now make explicit.

Proposition 2.5 ***Maximally extractive equilibria can include fair division***. *If $w > \frac{\beta_1(1-p_1)}{2+\beta_2/\alpha_2}$, then in any peaceful, Pareto-optimal settlement, both A and B have expected utilities strictly greater than their expected utility from war.*

Fearon (1995) tells us that if A has proposal power, he will leave B with just enough to avoid war. This is not the case in our model. We illustrate this in Table 1. Notice that B's utility from war is 1, but A's maximally extractive offer leaves B with 1.26. Why would A share the surplus? The answer lies in A's credible threat of war. The offers are sustained because issues are linked by each state's credible threat of war. If A takes too much of the surplus, he cannot credibly threaten to reciprocate. A makes a larger offer not because he values fairness, but because he must leave himself with a small enough fraction of the second issue that he can credibly threaten to fight.

Because issue linkage requires credible threats, we can establish another result.

Proposition 2.6 ***Issue linkage through threats of war has a lower interest bound.*** *If players do not value* *their least-favourite issue enough (i.e $\mathcal{C}_1$ does not hold) then only stable settlements can be supported in equilibrium.*

If B cares too little about issue 1 (β_1), then it cannot credibly threaten to expand a war should A threaten issue 2. Consequently, the only peaceful settlements would be stable ones, as defined in lemma 2.1. Pareto-improving settlements, such as the welfare-maximizing equilibrium in lemma 2.3, would be infeasible except in the trivial case where $w \geq \max(\frac{\beta_1}{1-p_1}, \frac{\alpha_2}{p_s})$.

This result clarifies results from issue linkage theory. Typically, liberal institutionalists suggest that efficient exchange is most likely when states care a lot about one issue and little about another (Keohane 2005). Our analysis shows that this is not true if issue linkage is held together by reciprocal threats of war. In this case, states must care enough about the issue they trade away in order to make their threats credible, should peace fail.

This threshold condition holds important empirical implications. It might explain why Great Powers often leave seemingly-ideal bargaining chips off the table. For example, the United States never raised Soviet control over East Turkestan during negotiations at Yalta, even though the Soviets cared substantially about it and would have been willing to make concessions to secure their policy in the region (Wang 1999). Why did the US not use East Turkestan as bargaining leverage? Our model suggests a simple answer: the United States did not care enough about East Turkestan to fight for it, and therefore could not credibly exploit it for exchange.

3 Re-negotiation and the risk of accidental war

In our baseline model, states always preferred efficient, contagious settlements over inefficient, stable settlements because nothing triggered the contagion process. In practice, much can go wrong in the decades after a peace conference concludes. Great powers usually negotiate over issues that extend beyond their sovereign borders. Over time, local populations can start insurgencies, refuse to pay taxes, or seek independence. This creates unanticipated local conflicts that can cause the settlement to break down over any one issue.[15]

[15]The model we present is a static model. Clearly, a problem with any conference settlement is that the the world changes. An emergent great power could demand a stake in one of the issues at hand, or one of the original great powers could experience a revolution that fundamentally alters their preferences or abilities, as France did in 1789. We leave these possibilities to dynamic models.

The most famous example is the spark in Serbia that ignited World War I. In 1913, the great powers did not know that the July Crisis would come *with certainty*. However, they did assess that Europe was tense. They understood that the rise of nationalism would likely trigger a crisis somewhere in Europe at some point in the future (Röhl 1994, 162-189). Further, they all acknowledged that once a crisis did start, the settlement that sustained peace between them was likely to collapse. One might wonder: given that the risk of an accident was high, and that an accident would cause a contagious conflict, why didn't the great powers re-negotiate a more stable settlement?

To answer this question, we extend the baseline model. We assume that states have pre-negotiated an efficient, contagious settlement $\tilde{q}$. We then introduce the risk of an accidental war and the possibility of re-negotiation. Define the re-negotiated settlement as $q^r = [q_1 = p_1, q_2 = p_2]$. Notice that $q^r \in q^\dagger$, i.e. q^r is stable. Define two independently drawn random variables $\psi_1, \psi_2 \in (0, 1)$ as the risk Nature plunges the respective issue into conflict.[16]

As before, we assume that all issues start in a state of peace ($\omega = \emptyset$) and that states are aware of the pre-negotiated settlement $\tilde{q}$. Then, the game unfolds as before, except that, before players decide whether to accept the settlement, the following occurs:

1. Players vote whether to attend a re-negotiation conference.
    - iff all vote to attend, Nature imposes settlement q^r upon them. Otherwise, $\tilde{q}$ remains the settlement.
    - For each issue i, Nature forces players to declare war against each other with probability ψ_i, but otherwise sustains peace.

First we characterize the conditions under which both states agree to either re-negotiation or not.

Proposition 3.1 *If either the risk of an accident over any one issue is high, or the cumulative risk of an accident over both issues is high, or the cost of war is high, then states agree to renegotiate. If instead these parameters are low, both states prefer to risk war contagion and do not want to renegotiate.*

We report the analysis in Appendix B. Players' decisions to renegotiate follow a simple utility comparison. This comparison weighs the benefits from efficient exchange against the risks of war contagion, should

[16] Incomplete information generates a similar result. We model an accident because we study a settlement that was in place for a long time.

Nature spark an accidental conflict. When the risk of an accident is high (low), both want to renegotiate (retain the efficient bargain).

The model reveals that states do not always share a preference for re-negotiation. When the risk of an accident is moderate, one calls for renegotiation but the other refuses because she prefers to profit from efficient exchange even at a moderate risk of war contagion. We use comparative statics to characterize the equilibria when players disagree. The formal analysis is in Appendix B. Here, we summarize the results.

Proposition 3.2 *Great power preferences to re-negotiate world order diverge, leading one state to demand renegotiation and the other to refuse, if:*

- *The initial settlement is closer to extractive (Lemma 2.4) than welfare-maximizing (Lemma 2.3). The state that profits the most from extraction refuses re-negotiation.*
- *One state has balanced preferences over the two issues, but the other cares a lot more about one issue than the other (e.g.* $\alpha_1 - \alpha_2 >> \beta_2 - \beta_1$*). The state with divergent preferences (A in this example) refuses re-negotiation.*
- *One state is stronger than the other on both issues. In this case, the weaker state refuses re-negotiation.*

To be clear, this analysis emphasizes pre-war re-negotiation given that an accident is possible. We emphasize this form of re-negotiation because it matches the puzzle of World War I, among other notable cases. In Appendix B.1, we allow states to re-negotiate one issue peacefully while fighting a war over a second issue. Allowing for frictionless re-negotiation once a war has started could cause all efficient exchange to unravel because states can initiate wars in one region and offer a compromise in the other. Moreover, we cannot find a historical case in which two belligerents re-distributed issues that were still at peace in order to prevent a war from spreading.

Appendix B.1 explains why re-negotiation does not compromise our result. First, we study a model where states can re-negotiate once war has started, but are uncertain about what their proposal power will be. In this case, they prefer efficient peace rather than local war with re-negotiation. The reason is that war and a settlement must fit within Fearon's bargaining ranging, forcing states to take inefficient deals in expectation. Second, we theorize that states pay a cost if they re-distribute an issue at peace while fighting an all-out war in a different theatre. Our results obtain if re-negotiating an issue at peace is sufficiently costly.

Finally, if we allow any state to expand war at any moment (and not only after they are attacked) then any equilibrium that sustains peace is re-negotiation proof following a local war.[17]

4 Multipolarity, complexity and war initiation

Waltz argued that polarity, the number of great powers in the international system, fundamentally affected the risk of war. Looking at the historical record, he noted that bipolarity produced simple agreements and few major conflicts. In contrast, multipolar worlds produced complex agreements and more conflicts. Observing this trend, he intuited that more complex systems were more prone to conflict initiation and war contagion.[18]

This argument sparked fierce debate. Hopf (1991) shows that multipolar systems are often peaceful. Others disagree (see Midlarsky and Hopf 1993). Waltz himself wondered why forward-looking states do not realize that starting a small war would cascade into a large one. He conjectured that states initiated local wars because they irrationally did not fully comprehend the chain reactions they would trigger.

We now extend our model to three players to better understand complexity in multipolar systems and address the puzzle of rational war initiation in multipolar worlds. First, we make precise what Waltz (1979) meant by multipolar complexity. That is, we contrast the agreements that sustain peace in bipolar and multipolar worlds. We argue that multi-polar systems allow for more complex threats of war contagion. We conjecture that complexity is difficult to define informally, or measure empirically, because threats of wartime contagion are usually unobserved. But that does not mean that complex threats do not have important affects. We show theoretically that the anticipation these complex threats in multi-polar worlds sustain agreements that cannot be sustained in bipolar worlds.

Second, we show that complexity features of multi-polar agreements can also explain rational war initiation with complete and perfect information. That is, it is possible to forge peace agreements that accept some chance of world peace, some chance a local war breaks out, and some chance the system collapses into a multi-actor, multi-issue conflict.

[17]Consider a sub-game in which both states initiate war over issue 2 if a war starts over issue 1. Since both states launch war there is no profitable deviation to no-war.

[18]Waltz argues that in multipolar worlds ”Dangers are diffused, responsibilities blurred, and definitions of vital interest easily obscured... To respond rapidly to fine changes is at once more difficult, because of blurred responsibilities, and more important, because states live on narrow margins. Interdependence of parties, diffusion of dangers, confusion of responses: These are the characteristics of great-power politics in a multi polar world” 1979, 171.

4.1 Setup

We extend the model to three players, $j \in A, B, C$, who bargain over two issues each (i.e. six bilateral bargains). Adding a third player generates considerable complexity both to strategies and to notation. To deal with this complexity, we assume that in every bilateral relationship each player values one issue high $H = 1$ and the other low $L \in (0, 1)$. We assume players have complementary preferences (e.g. what A values high B values low). In the bipolar model, we would express this as $H = 1 = \alpha_1 = \beta_2 > \alpha_2 = \beta_1 = L$.

We denote a specific good, associated offers, and war choices with subscripts mn, where actor m values the good high and n values the good low. Accordingly, we define q_{mn} as an offer over the good mn in which m gets q_{mn} and n gets $1 - q_{mn}$. For example, issue AC is valued High by A and Low by C; player A gets q_{AC} in peace and wins a war over AC with probability p_{AC}. The same players also contest CA, which C values high and A values low; A gets $1 - q_{CA}$ in peace and wins war with $1 - p_{CA}$. The following condition will sometimes be helpful to make the analysis tractable; we will indicate below when we employ it:

$$\mathcal{C}_2 : p_{AC} = 1 - p_{CA} > p_{AB} = 1 - p_{BA} > \frac{1}{2} \text{ and } p_{AC} > p_{BC} = 1 - p_{CB} > \frac{1}{2}$$

$\mathcal{C}_2$ requires that p obey a simple linear ordering of players' strength; informally, it requires $A > B > C$.

We re-define a settlement as a vector $q = [q_{AB}, q_{BA}, q_{BC}, q_{CB}, q_{CA}, q_{AC}]$. We re-define ω, the state variable that records what issues are at war, to include two letter indices (eg AB, AC).

The sequence of moves is identical to the baseline, bipolar game, except that step (2) is repeated until every issue is at war, or until no state takes the opportunity to declare war.

4.2 Defining Waltzian complexity in multi-polar worlds

Waltz and others (Keohane 2005) argued that multipolar agreements were more complex. But what did they mean by complex? We now study equilibria that can sustain peace in multipolar but not bipolar worlds. We argue that these agreements are more complex in terms of the threats of war contagion that sustain them. Specifically, the contagion pathways that sustain multipolar peace can include stochastic attacks or multilateral threats that drag third-parties into war.

We then show that these more complex threats have important implications for the negotiated settlements we observe. Settlements supported by complex threats defy what bilateral theories suggest is possible

(Joseph 2020). For example, one state will accept peace with another, even though she gets less than her minimum demand for fighting in that dyad. These two points are related. We can only support counterintuitive distributions when contagion would drag third-parties into conflict or include stochastic elements.

What do we mean by complex threats? Table 2 visualizes three threats of war contagion that can peacefully sustain some offer in a multipolar world. Row 1 visualizes reciprocal (i.e. within-dyad) threats of war. In the bipolar world all efficient agreements relied on reciprocal war-contagion (Proposition 2.2). In the multipolar world, reciprocal contagion also sustains many efficient equilibria.

Rows 2 and 3 depict threats that can sustain some peaceful offer in a multipolar world but not a bipolar. The threats in rows 2 and 3 are more complex than reciprocity (row 1) because they increase the number of states and issues that are dragged into war once war starts. By involving more actors and issues, these complex threats also expand the scope of war contagion.

Row 2 presents a chain-reactive threat. It sustains an efficient settlement by dragging a third party into the dispute. A is considering attacking B. In equilibrium, if A attacks B, B attacks C, and then C attacks A. A is deterred from attacking B because of the threat imposed by C.

Row 3 presents a contingent threat. A is considering attacking B. B's response is stochastic. With some probability, B reciprocates against A. Otherwise, B attacks C, setting off a chain reaction in which C attacks A. C's response is contingent. C only attacks A if B attacks C first.

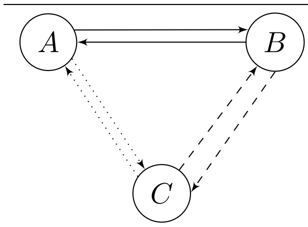
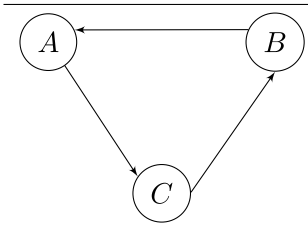
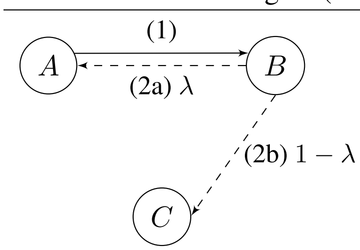
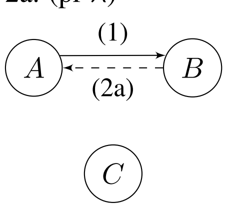
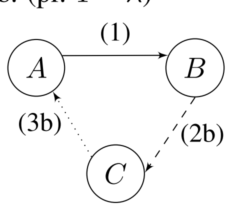
More complex threats expand the scope of feasible, Pareto-dominant peaceful bargains. Because these threats spread conflict to additional actors and additional issues, states can construct more extractive and more efficient offers than are possible with simple reciprocity. Understanding the often-tacit threats sustaining multipolar settlements is vital to understanding the peace that emerges.

Proposition 4.1 *In a bipolar world, agreements that sustain peace are bounded by each state's minimum demand for war over two issues. In a multipolar world, we can sustain peace with agreements in which A extracts so much from B that B prefers to fight two bilateral wars against A.*

Proposition 4.2 *In a bipolar world, agreements that sustain peace are bounded by each state's credible threat of retaliation. In a multipolar world, we can sustain peace with agreements in which A extracts so much from B on both issues, that A cannot threaten to reciprocate if B initiates a conflict over either issue.*

Propositions 4.1-4.2 are existence claims. We can validate both with a single example that we describe

Table 2: Waltz' Complex Threats: Threats of war contagion that can sustain peace in multi-polar worlds.

Visualization	Example	States at war	Local conflicts
Case 1: Reciprocal Threats (possible in bipolar)			
	If A attacks BA, B attacks AB	2/3	2/6
Case 2: Chain-Reactive Threats (not possible in bipolar)			
	If A attacks CA, C attacks BC, and B attacks AB	3/3	3/6
Case 3: Contingent (Stochastic) Threats (not possible in bipolar)			
	Step 1: A initiates war against B on BA. Step 2a: B reciprocates by attacking AB (prob, λ). Step 2b: B attacks C at CB (prob $1-\lambda$)		
2a. (pr λ)  2b. (pr. $1 - \lambda$) 	Step 3a: Given 2a, the war stops expanding. Step 3b: Given 2b, C attacks A at AC.	(2 or 3) /3	(2 or 3) /6

Notes: Each panel visualizes a pathway for war contagion thathttps://www.overleaf.com/project/5fd61b81729535f0870155cb is part of an equilibrium that sustains an efficient peace. Arrows represent the direction of contagion. In case 1, line-types represent common contagion pathways. In case 3, line-types represent the sequence of attacks.

in Table 3.[19] Consider a world in which C is the weakest state, A is the strongest, and B is in the middle. Specifically, let $p_{CA} = \frac{1}{12}, p_{CB} = \frac{1}{6}, p_{AB} = \frac{3}{4}$. Take C's perspective and ask this question: for a specific set of punishment strategies, what peaceful settlement will maximize C's utility?

Column 2 of Table 3 assumes that states will play reciprocal punishments. As it shows, the offer that maximizes C's utility is equivalent to what C would get from two maximally extractive bilateral bargains. Thus, adding in a third player yields no extra insights if we assume that threats of war contagion are reciprocal.

Column 1 of Table 3 assumes that peace is supported by a more complicated set of threats for war contagion. (For complete description, see Appendix C.1.) It includes a chain-reactive threat: if A attacks C over CA, C expands the war to BC, and B, in turn, expands the war to AB and BA.

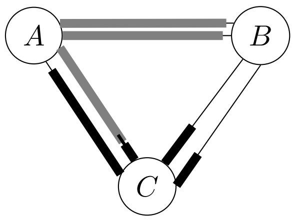
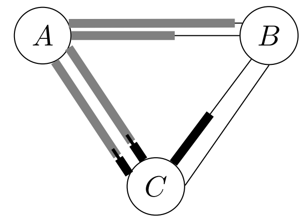
As described in proposition 4.1, player C exploits this chain reaction to extract more of the surplus than what reciprocity allows. C effectively 'hires' B to make coercive threats against A. Even though A is stronger than C, A knows that war against C will will trigger B to attack A. Thus, C can exploit B's strength, and B's credible threat, to extract more of the surplus from A.

As described in proposition 4.2, C has extracted so much in its relations with A that it cannot threaten war against A, even if A attacks CA and/or AC. In the bipolar world, Lemma 2.4 makes this situation impossible. It is possible in the multipolar world because C has left itself a credible threat against BC, which will in turn trigger war between A and B.

In Appendix C.1.1 we generalize these results by showing that reciprocal threats of war cannot sustain offers that contain the features described in proposition 4.1-4.2. We reinforce our finding that multi-polar systems can generate offers that seem to defy standard bargaining results because they facilitate uniquely complicated threats of war. However, it is also possible to extend the logic of bipolar efficiency (and also stability) to multipolar systems. Later, we'll use this insight to explain some puzzling empirical heterogeneity in seemingly similar settings.

[19] To be clear, this is not the only example: focusing on extreme settlements, or violating $\mathcal{C}_2$, will further exaggerate what a player is able to extract. We focus on this example because it obeys strict limits on p_{ij}; because it shows how even a very weak state (C) can extract a surprisingly large share of the pie from her rivals (A and B); and because it demonstrates counterintuitive strategies a diplomat might employ. We provide technical details in Appendix C.1.

Table 3: Visualizing the most C can extract given different threats of war contagion

Complex Threats, including chain-reactions		Reciprocal Threats	
Offers	C extracts		Offers
	$q_{CA} = 1, 1 - q_{AC} = \frac{1}{6}$ $q_{CB} = \frac{11}{36}, 1 - q_{BC} = \frac{1}{60}$	$q_{CA} = \frac{2}{9}, 1 - q_{AC} = \frac{1}{6}$ $q_{CB} = \frac{26}{36}$	
	Total utilities		
	A: 2.43 B: 1.4 C: 1.42	A: > 2.33 B: > 1.55 C: ≤ 1.04	
	Contagion if war initiated over		
	$CA \to BC \to AB$ and BA	$BC \to CB$	

Other parameters: $p_{CA} = \frac{1}{12}, p_{CB} = \frac{1}{6}, p_{AB} = \frac{3}{4}, L = \frac{3}{5}, w = \frac{1}{12}$

Notes: Pictures visualize what C can extract in peace given a specific punishment strategy. This is what C would have offered in an ultimatum bargain. The thick black segments represent C's share of an offer. Larger thick black lines mean C gets more in peacetime. Grey segments represent A's share. Thin lines are B's share.

4.3 Rationalist war initiation with complete information.

So far, we have focused on settlements where no state wants to initiate war. Thus, war only occurred when we allow for an accident (section 3). We now use the logic of complex peace to provide a rationalist logic for war initiation in multipolar worlds. We call these *volatile* equilibria. In them, no player initiates war with certainty, but at least one player initiates war with positive probability.

Volatility is distinct from four other concepts. First, unlike unsustainable settlements, volatile equilibria include some chance of global peace, i.e. all players accept the settlement. Second, unlike peaceful settlements, volatile equilibria include some chance of war initiation. Third, unlike contagion, volatility describes a settlement with a strategic choice to initiate war, whereas contagion describes the risk that a local war spreads after it begins. Finally, unlike accidental conflict (section 3), volatility requires a conscious choice to initiate war knowing that contagion could follow.

Volatile settlements exist under a variety of conditions. The proposition below outlines a non-exhaustive list of plausible sufficient conditions for their existence.[20] Note that these conditions are fairly lax — e.g. the floor for w never exceeds $\frac{1}{8}$ — and so volatile equilibria are possible in a wide variety of real circumstances.

[20] See appendix C.1 for examples which violate one or more of these conditions.

Proposition 4.3 *A volatile equilibrium must exist if the following hold:*

$$\mathcal{C}_1 \textit{ and } \mathcal{C}_2$$

$$p_{BC} \leq p_{AB}$$

$$L \leq \frac{1}{2}$$

$$w > \max\{\frac{1}{3}p_{BA},\ p_{AB}\frac{L}{2L+1}\}L$$

In a volatile equilibrium, each of the following has nonzero probability: global peace, a non-contagious war that involves two actors fighting over a single issue, and contagious war that includes many actors and issues.

In volatile settlements, one state (for example, A) extracts an enormous amount of the surplus from the others (B and C). A extracts so much from B and C that A cannot credibly threaten retaliation if any one opponent initiates war against him. However, the amount A extracts is low enough that if both B and C simultaneously attack A, then A can threaten to retaliate. Furthermore, A's retaliation triggers a contagious war that plunges multiple territories into conflict.

B's best outcome is realized if B attacks A leading to a local war in a single theatre. B's worst outcome is realized if B and C simultaneously attack, leading to global war contagion. Thus, whether B profits from initiating war depends on what C does. B's middle outcome is that B does not initiate war. Critically, if B does not launch a war, B's utility does not depend on C's action. C has an equivalent preference order.

In the end, B's and C's best strategies are to keep each other guessing by initiating war with nonzero probability. This creates some probability of global peace, some probability of a local war, and some probability of a global conflict.

In theory, a volatile equilibrium could unravel if B and C could coordinate so that only one launched an attack. The trouble is, they do not want to coordinate with each other. Both want to challenge the status quo, and neither will stand down if his promise to fight would be believed. In this way, the extractive state (A in this example) can keep other actors in check only if it exploits all of them a large amount. Had A taken a huge amount from B but not C, then B would have initiated war with certainty because B knows that C will not attack.

To be clear, volatile equilibria are usually (though not always) Pareto-dominated by a peaceful equilibrium. Nonetheless, we believe that volatile world orders are plausible for several reasons. Below we describe two rationalist reasons. We report technical examples in Appendix C.2.

First, the possibility of an accidental crisis can make highly extractive, volatile settlements more attractive than less extractive, more stable equilibria. In a volatile equilibrium, the exogenous risk of a crisis can be absorbed without reducing A's overall payoff. For example, if there is an exogenous hazard ψ_{AC} of accidental conflict over AC, then C will simply reduce the probability it attacks AC by the same amount, leaving A's volatile payoff unchanged. By contrast, in a non-volatile settlement, the hazard ψ_{AC} would necessarily reduce A's expected payoff. When the risk of an accident over a specific issue is sufficiently high, certain volatile equilibria leave the extractive state better off than other world orders.

Second, a rational leader with a short time horizon might prefer a volatile settlement to a peaceful one. If we consider the universe of settlements that can end in global peace, the most extractive are almost always volatile. Because the costs of volatility will accrue only in the long run, whereas extracted rents will begin flowing immediately, a leader might sacrifice the long-term interests of his nation to present a domestic audience with an immediate short-term gain.

The logic of volatile settlements complements Waltz (1979)'s broader argument. Waltz was puzzled why states could not anticipate the far-reaching consequences of the local wars they started. From the war initiator's perspective, war could lead to either an undesirable, global war through a contagion process, or a desirable local war. Consistent with Waltz, we argue that miscalculation can arise in multipolar worlds because the consequences that follow from war initiation depend on what else is happening in the system. When there are many aggrieved states, one initiator cannot control nor perfectly anticipate another's choices. Many states, each with unique incentives to initiate war, create the possibility for catastrophic war. We go one step further by explaining why states would agree to live in this world. At least one state may construct a volatile peace that risks highly contagious war because it allows him to extract a surprisingly large amount from peace—so long as peace lasts.

5 Implications for two other research agendas

Above, we emphasized the model's implications for issue linkage and for polarity and conflict. Below we explain how our theory contributes to two other research agendas. We will illuminate aspects of these contributions in our case material.

5.1 War contagion

Scholars of war contagion argue that specific connections between states determine the ways that wars expand (Levy 1982). Careful historical work shows that the pathway for war contagion follow alliances in some cases (Christensen and Snyder 1990). But in other cases, alliances seem less important. Instead, contagion follows commercial/technological interests (Barbieri and Levy 1999; Bas and Coe 2012), geography (Mearsheimer 2001), spheres of influence or other institutional features (Braumoeller 2008; Lake 2009) or other pathways (Vasquez 2018). Are all of these theories correct? If they are, why are some pathways important in some cases and not in others? Furthermore, contagion theorists often struggle to fit the dogs that don't bark. There are many historical examples where states held interconnected relationships but their limited wars did not spread. For example, Britain and France had extensive trade ties and complex alliance relationships in the 18th Century. And yet, the Fashoda Crisis did not expand into a contagious conflict.

Our theory resolves these puzzles in a way that complements existing results. The dogs that don't bark can be explained because contagion is most likely if states efficiently distribute territories when they craft world order. Focusing on world orders that efficiently exchange territory, we explored the different pathways along which contagion could unfold. We showed that similar settlements could drive many different contagion pathways if peace collapsed. This could explain why contagion sometimes follows alliances and other-times follows trade or geography.

5.2 Diplomacy as peace-time negotiation not information in a crisis

Scholars extensively study how rationalist diplomacy reveals information in a crisis (Malis and Smith 2020; Sartori 2005; Kurizaki 2007; Trager 2015; Brutger 2021). But diplomacy has many functions. Historians believe that diplomats plays a pivotal role as negotiators during major international conferences (see Trager 2016; Brutger 2021). However, careful game-theoretical work down-plays the importance of negotiations because bargains are largely determined by protocols and outside (or inside) options (Fey and Kenkel 2020; Muthoo 1999).[21] If protocols and exit options determine the outcome, then why do historians classify diplomats based on their negotiation skills? We argue that during conference negotiations clever diplomats do more than exploit protocols to set the size of offers. They also manipulate the punishment strategies if peace fails (Slantchev 2005). A skilled diplomat will create tension between third-party countries to provoke

[21] This does not discount important psychological work (eg Yarhi-Milo 2014).

threats of war contagion should war start. She will then constructs offers that extract a lot for her state, while incentivizing others to follow through on specific threats. We go one step further by showing that favourable settlements have negative consequences because they raise the risk of war contagion.

If extractive equilibria represent diplomatic skill, then our model predicts that clever diplomats are partially responsible for war initiation and catastrophic war contagion in multipolar worlds. This has normative implications that we partly explore through examining the divergent types of war in the cases below. For example, some historians venerate Metternich because he constructed a very complicated world order that advantaged Austria. But what they may not have considered is that Austria suffered more in war because the collapse of world order was devastating.

6 Empirical Illustrations: The Treaty of Aix-la-Chapelle and Peace of Utrecht

Our theory generates several empirical implications. We cannot rigorously test them all in one article. Instead, we focus on illustrating the key mechanism that drives our results: the stability-efficiency trade-off. Our theory predicts that if states live under a world order that efficiently distributes issues and territories, then conflicts that erupt between a few states are likely to devolve into multi-issue, multi-state wars.[22] Conversely, when states live under a world order that inefficiently distributes issues and territories, then conflicts that erupt between a few states will not spread to other states or issues.

Our independent variable is the amount of efficient exchange we observe in how states distribute issues and territories when they forge world order at major international conferences.[23] We measure efficiency in two ways. First, we use historical accounts of elite deliberations. In our theory, efficiency is possible because states value issues differently, and so they can benefit from exchange. Therefore, a settlement is more efficient when states gain the issues about which they care about most, and less efficient when they divide all issues between themselves. If historians identify obvious exchanges that states could have made but did not, or if elites express frustration because they wanted to make a trade but did not, then it suggests the agreement is inefficient. If elites from different countries acknowledge that they received the issues they cared most about, then the settlement is more efficient.

Second, we exploit the fact that states negotiate world order in the wake of major wars that ended in

[22] We showed that multipolar worlds allowed for very complex settlements, they can also sustain stable, non-contagious settlements.

[23] We define efficient versus inefficient in section 2.1 and explore the various forms of efficiency in Lemmas 2.3 2.4.

partial stalemates. When negotiations commenced, each army controlled some territory in many different theatres. In the cases we study, the military situation offers a convenient place to draw a boundary that reflects power realities but does not reflect state preferences. In contrast, and as we shall see in the historical record, the purpose of exchange is to get more out of peace. We argue that drawing borders based on the military situation is usually less efficient than swapping territories during post-war conference negotiations.[24] The latter offers a more efficient exchange.

Our dependent variable is the level of contagion that follows once a local conflict starts. To evaluate contagion, we ask—when a small conflict broke out, did the conflict remain concentrated between two actors over a single issue, or did it expand to include more actors and issues?

Our central prediction is that bilateral conflicts that start under an efficient world order are likely to expand to many belligerents and theatres of conflict. In contrast, crises that start under an inefficient world order will not spread. Over time, many bilateral crises may erupt in a stable but inefficient world order, and we expect that none of them will devolve into major war.

We examine two peace agreements: the Peace of Utrecht (1713-1714)[25] and the Treaty of Aix-la-Chapelle (1748). We chose these cases for two reasons. First, as Vasquez (2014) argues, the Great Power dynamics in Europe before 1945 offer fertile ground to build and test theories of the causes and scope of war. Positivist political science usually focuses on the World Wars (most recently Debs 2020), which Copeland (2000) argues are over-determined. Instead, we chose two understudied cases that leverage the European system and which international relations scholars and diplomatic historians believe are critical for the development of the European international system (Ikenberry 2001, p40-44; Paul 2018, 8 Scott 2006).

Second, the two settlements share a number of similarities that make them a good comparison with each other. Both followed a major European war, the War of the Spanish Succession (1701-1714) and the War of the Austrian Succession (1740-1748) respectively. They are reasonbly proximate in time. Both took place in the same strategic context: one side had the upper hand but had failed to gain a decisive victory, and all parties were financially exhausted. Moreover, the treaties mainly involved the same actors: Britain, Austria, the Dutch Republic, France, Spain, Savoy, and Prussia.[26] Finally, the treaties also followed extensive formal negotiations at an international conference dominated by the great powers.

[24] This is especially true if belligerents do not exchange territory in any theatre. Of course, if one state keeps one territory it conquered but otherwise engages in exchange, it would likely reflect efficient exchange.

[25] This refers to the treaties of Utrecht, Baden, and Rastatt.

[26] Russia was a party to the Treaty of Aix-la-Chapelle. There were also changes among smaller states.

Despite these similarities, the aftermaths of the treaties followed radically different trajectories. Following the Peace of Utrecht, no general European war broke out for another 26 years. This may seem short by today's standard, but it was extraordinary at the time, considering that Great Powers fought major wars for 29 of the 40 years preceding the treaty.[27] There were several limited conflicts, but none escalated into a general war. In contrast, the Treaty of Aix-la-Chapelle secured only eight years of peace, and its first crisis escalated into the multi-party, multi-theater Seven Years' War (1756-1763).

6.1 The Peace of Utrecht

In 1700, Charles II (the Bewitched), the last Habsburg ruler of Spain, died childless. Two candidates claimed the throne: Philip of Anjou, grandson of Louis XIV of France, and Charles of Habsburg, the second son of the Austrian Emperor Leopold I. Their dispute over the Crown quickly escalated into a general war that involved five great powers. France and Bavaria supported Philip and Austria, while the 'Grand Alliance' of Great Britain,[28] the Dutch Republic, and later Savoy supported Charles (Clark 1970). By 1711, the Grand Alliance had the upper hand after capturing Spain's possessions in Italy, the Spanish Netherlands, and Bavaria. However, they had been unable to invade France itself, and Charles had been ejected from Spain (Lynn 1999, 266-360).

After more than a decade of conflict, all parties were exhausted. It was increasingly clear that neither would achieve a decisive victory. Accordingly, a compromise peace became inevitable.

6.1.1 Peace Settlement

Since the War of Spanish Succession involved many belligerents fighting in many theatres, the Peace of Utrecht was multi-faceted. The following summarizes the main terms of peace. The main result was that Philip became King of Spain and its overseas empire, while Charles, who had become leader of Austria and Holy Roman Emperor after the death of his brother in 1711, received most of Spain's other European possessions. These included the Spanish Netherlands (roughly corresponding to modern-day Belgium and Luxembourg), Milan, Naples (the southern part of mainland Italy), and Sardinia. Philip's receipt of Spain reflected the Franco-Spanish victory on the Iberian Peninsula. However, the settlement prohibited Philip and his successors from ever gaining the French throne, which was an essential British demand (St. John 1809,

[27] The Franco-Dutch War (1672-1678), the Nine Years' War (1688-1697), and the War of the Spanish Succession.

[28] Formally England until the 1707 Act of Union.

143). Similarly, the Grand Alliance had prevailed in the Italian theater, primarily due to Austria's efforts. Otherwise, Savoy received Sicily for its contribution to the war in Italy. France kept Alsace, which it had gained in 1684 and held during the war. Britain kept strategically important outposts it had captured, most notably Gibraltar and Menorca. It also retained territory that allowed for the highly profitable monopoly of exporting slaves to Spanish colonies in the Americas (Pitt 1970). Only two points significantly diverged from the military situation: France ended its occupation of Lorraine, and Austria ended its occupation of Bavaria. The conditions of the settlement did not leave any party especially happy. For instance, Austria wanted Charles to gain all of Spain, while the Dutch wanted full control over the Spanish Netherlands (St. John 1809, 122).

We code the Peace of Utrecht as inefficient for two reasons. First, the settlement largely reflected the military situation at the end of the war. Second, historians have pointed out several exchanges that could have left both parties better off.

The settlement regarding the Spanish Netherlands illustrates the agreement's inefficiency. First, the Spanish Netherlands had been conquered with English, Dutch, and to a lesser extent Austrian soldiers during the war. While Austria formally received the territory, in practice, Vienna had to share it with the Dutch Republic. The so-called Barrier Treaties gave the Dutch the right to garrison seven key strategic fortresses. Moreover, Austria had to cover 60% of the Dutch garrison costs (Pitt 1970, 476-479). Such shared sovereignty reflects the issue-splitting we identify as a hallmark of stable (but inefficient) peace agreements.

Second, Austria did not particularly want the Spanish Netherlands. The territory was far away from the Austrian heartlands on the Danube, leaving it difficult to defend against any renewed French aggression (Hochedlinger 2003, 222-225). Moreover, historians have long pointed out an obvious alternative. Austria could have exchanged the Spanish Netherlands for the more proximate and defensible Bavaria (Holboern 1982, 113-114 Simms 2007, 66). In return, the Elector of Bavaria would have gained a more populous and prosperous realm, one which he had already acted as a governor of during the war. To sweeten the deal, the Elector could gain the title of king he coveted. The historian Hochedlinger argues that such an exchange "would have significantly benefited both sides. Austria would have consolidated her heartlands, while the House of Witteslbach, thus removed from the immediate grip of its Austrian rival, would have regained more freedom of action" (Hochedlinger 2003, 187). This is not idle speculation: such an exchange was a key goal for Vienna until the French Revolutionary Wars (1792-1802).

Other aspects of the Peace of Utrecht were similarly inefficient. Austria and Savoy would have benefited from exchanging Sardinia for Sicily. Austria, which lacked a navy, had no hope of defending Sardinia. However, Sicily and Naples together would have formed a stronger defensive front if united. Conversely, Savoy would benefit from the more proximate Sardinia. Such an exchange did indeed take place in 1720, following the War of the Quadruple Alliance (Shennan 1995, 25).

6.1.2 Crises and war expansion

Consistent with our theory, the inefficient Peace of Utrecht prevented another general, great power war for 24 years. This was a very long period of peace for the time. The robustness of the settlement is surprising in light of structural changes taking place throughout these years, such as the gradual resurgence of France and the rise of Prussia.

To be clear, several military crises did cause local wars between 1718 and 1738 (McKay and Scott 1983, 94-158). However, consistent with our theory, none of these small wars spread. Furthermore, the basic contours of world order persisted after the conclusion of these local wars. For example, in 1718 a resurgent Spain retook Sardinia almost unopposed. The following year, it also invaded Sicily. However, the other European great powers united and forced Spain to sue for peace in the 1720 Treaty of the Hague. The main result was that Austria exchanged Sardinia for Sicily with Savoy (Blanning 2007, 563-564). In 1727, war broke out between Britain and Spain over Gibraltar. However, the war ended two years later with a return to the status quo ante, and it did not involve other states (Simms 2007, 212).

The most serious challenge to the Peace of Utrecht came with the War of the Polish Succession, which broke out in 1733. The pretext was a French attempt to install its candidate on the Polish throne against the wishes of Austria. However, France's true aim was to weaken Austria and gain Lorraine (Sutton 1980). This case included two contagion steps, in that Russia also supported Austria's candidate and Spain provided military assistance to France to increase its influence in Italy. However, while Great Britain, the Dutch Republic, and Prussia were concerned about a resurgent France, there was no further contagion. Furthermore, the war did not expand to all potential issues between France and Austria for the strategic reasoning that our theory expects: France understood that attacking the Austrian Netherlands would have dragged the British and Dutch into the war. Paris viewed this as too costly given the bargain it held (Black 1986). Accordingly, there was no fighting in the Low Countries, despite this being the only direct border between France and Austria.

The War of the Polish Succession ended with the 1738 Treaty of Vienna, which made some small modifications to the Utrecht settlement without fundamentally changing its terms. Austria lost Sicily, Naples, Parma, and Piacenza to the younger sons of King Philip V of Spain. Lorraine went to the father-in-law of the French King Louis XV and would become French at his death. As a compensation, the Duke of Lorraine (and husband of future Austrian Empress Maria Theresa) gained the Duchy of Tuscany (Sutton 1980, 191-210).

In summary, the Peace of Utrecht largely reflected the military situation at the end of the War of the Spanish Succession. Moreover, historians have noted opportunities for efficient exchanges that the parties did not take. Thus, we code the settlement as inefficient. In the wake of this agreement, European order was remarkably durable. Several local disputes escalated to bilateral conflicts. Nevertheless, no bilateral conflict escalated into a general war. Most remained limited to one issue and resulted in only minor changes to the overarching settlement.

6.2 The Treaty of Aix-la-Chapelle

The Utrecht settlement finally broke down in 1740, when Emperor Charles VI of Austria died without any male heirs. Under the Pragmatic Sanction, Charles secured international recognition for his daughter Maria Theresa inheriting all of his possessions (Ingrao 1995, 129-134). Nevertheless, Prussia took advantage of his death to renege on its promise and seized the rich province of Silesia (Blanning 2016, 97-115). Austria's failure to regain the province then encouraged Saxony, Bavaria, and France to seek to partition Austria.[29] Britain initially sought to remain neutral, but finally had to intervene to prevent the collapse of Austria (Simms 2007, 274-304). The previous year, Anglo-Spanish colonial tensions resulted in the War of Jenkins' Ear. Eventually the wars in Europe and the Americas merged into a single global conflict (Scott 2006, 39-57).

Austria, with British and Russian help, prevented partition. But, with wars on several fronts, Austria could not retake Silesia from Prussia. By 1747, France had captured the Austrian Netherlands (Hochedlinger 2003, 259-262). In other theatres, Austria and Great Britain had more luck. Austria in cooperation with Sardinia (formerly Savoy) repelled French and Spanish invasions of Northern Italy and captured Genoa and Modena (Anderson 1995, 158-170). In Canada, Britain made numerous gains, most notably the strategically important fortress of Louisbourg (Simms 2007, 344-347). The war ended in 1748 with the Treaty of Aix-

[29] Exogenous shocks that threaten to remove one of the great powers from the system is outside the scope of our model.

la-Chapelle. The treaty was primarily the result of negotiations between Britain and France, which then imposed the settlement on their respective allies.

6.2.1 The Peace Settlement

We code the Treaty of Aix-la-Chapelle as efficient for two reasons. First, unlike the Peace of Utrecht, the Treaty of Aix-la-Chapelle involved several territorial exchanges that departed from the military situation at the end of the war (Anderson 1995, 193-209). France agreed to give up its conquest of the Austrian Netherlands in return for Louisbourg. In Italy, Austria and Sardinia agreed to return their conquest of Genoa and Modena and Austria agreed to cede Parma. The only significant element that did reflect the military situation was that Prussia kept Silesia.

Consistent with our definition of efficiency, the settlement better reflected the preferences of the different belligerents. Despite the increasing importance of North America, Great Britain's strategic priority remained keeping France out of the Low Countries (Simms 2007, 350-351). Conversely, France no longer held a strong desire to annex the Austrian Netherlands because it was now focused on overseas expansion. In that respect, it was essential to regain Louisbourg, which guarded the entrance to the St. Lawrence river (Broglie 1895, 73, Sosin 1957). Thus, both countries were happy to make the exchange.[30]

While Aix-la-Chapelle saw considerably more exchange Utrecht, some aspects of the settlement reflected the military situation. Notably, Prussia kept Silesia after taking it from Austria. However, this part of the settlement is clearly different (and more efficient) from the case in which Austria gained the Spanish Netherlands during the Peace of Utrecht. There are two reasons. First, Prussia strongly desired Silesia, a rich and contiguous province. In fact, Prussia viewed it as vital concession if it was to become a great power (Scott 2006, 68). Second, Prussia gained Silesia in full, without having to enter into a power-sharing arrangement or other mechanisms to split it. Neither condition applied to Austria gaining the Spanish Netherlands in 1713.

It is also notable that Austria was unhappy to permanently lose Silesia to Prussia. However, and consistent with our theory, the complex nature of the agreement allowed the great powers to keep this bilateral settlement in force by locating it within a larger architecture. Austria could not retake the territory without

[30]Many historians have argued that French gains were meagre in the light of its strong military position. However, there is no evidence that France at the end of the war desired to make wider gains. Indeed, Louis XV explained his actions by stating that he was "a king not a merchant" (Broglie 1895, 3). This, coupled with France's strong bargaining position, suggests that the treaty reflected French preferences.

British support, and Britain was not willing to give support under the terms of peace. Furthermore, Austria was confident that it would be able to regain Silesia in a future war, as long as it would not be distracted by a war with France (Hochedlinger 2003, 330-337). In fact, Austria started planning for this the following year. Austria's plan was a major reason for the outbreak of the Seven Years' War in 1756 (Mcgill 1971).[31]

6.2.2 Crises and war expansion

Unlike the Peace of Utrecht, the Treaty of Aix-la-Chapelle did not survive even its first series of crises. The complex way that peace unraveled reflects the contagion process that our theory predicts. In the Americas, both France and Britain remained determined to expand their hold over frontier areas, particularly in Ohio. This resulted in the outbreak of the French and Indian War in 1754 (Scott 2006, 77-80).

In Europe, Austria did not accept its loss of Silesia. Seeing France focus on North America, Austria sought to detach France from its alliance with Prussia (Mcgill 1971). Such a rapprochement led to increasing distress in London and Berlin, who responded by signing a neutrality treaty with each other in 1756 (Scott 2006, 90-91). Rather than securing peace, this agreement propelled Austria and France to sign a defensive treaty (Stollberg-Rilinger 2021, 401-410). Austria was also allied with Russia. Fearing an imminent attack on Prussia by a vastly superior coalition, Frederick the Great responded by invading Austria's ally Saxony, which led to the outbreak of general war (Szabo 2007, 36). With Prussia's survival in danger, Britain had to send an expeditionary force to the continent, merging the American and European conflicts. Thus, the first major set of crises following the Treaty of Aix-la-Chapelle escalated into the Seven Years' War, an even larger and more intense conflict than the War of the Austrian Succession.

The way war spread when the Treaty of Aix-la-Chapelle faced its first crisis mirrors our description of a highly contagious equilibrium with chain-reactive threats. By giving whole issues to a party, other states had little incentive not to escalate. These efficient (but unstable) bargains culminated in a major war just eight years later.

6.3 Alternative Explanations

The most prominent explanation for the longevity of the Peace of Utrecht is its reliance on the balance of power, which article II of the treaty explicitly mentions. Our model includes power variables and we accept that power is necessary for explaining peace. However, the reference to the balance of power in the

[31] While we do not have space to explore it, Austria's logic plausibly fits how we understand a volatile settlement.

treaty is vague, due to the many meanings of the term (Wight 1978, 168-185). Thus, the reference to the balance of power is in itself insufficient to explain the long peace. Furthermore, the main interpretation of the balance of power does not adequately explain why the settlement remained stable despite significant changes in the distribution of power over the next two decades. Nor is it clear how the Treaty of Aix-la-Chapelle, which did not prove stable, failed to conform to the balance of power. Our theory offers a solution to this shortcoming. We model how power, preferences, and threats interact to produce specific settlements, whose stability should vary in predictable ways.

Another explanation holds that that the European great powers, spent by decades of war, were unwilling to pursue further conflicts after the Peace of Utrecht. Dynastic difficulties also left key states weak and distracted by domestic politics. In Great Britain, the new House of Hanover faced significant opposition from Jacobites. In France, the new king Louis XV was a child and the only remaining heir after his grandfather, father, and older brother had died in quick succession. And In Austria, Charles VI lacked a male heir.

Financial exhaustion and dynastic problems help explain why war did not break out immediately, but they are not sufficient to explain the peace's longevity. The Utrecht settlement proved durable even after the great powers strengthened their finances and solved their dynastic problems. Moreover, all the belligerents were financially exhausted at the end of the War of the Austrian Succession, but their exhaustion did not prevent yet another war eight years later.

Because Europe was war-prone during this period, historians have largely taken for granted that the Treaty of Aix-la-Chapelle quickly devolved into major war. However, when set beside the Peace of Utrecht, it is surprising that the Great Powers did not seek to replicate Utrecht's stability. Since we provide one of the first social science theories that connects the qualitative features of peace settlements to the scope of war, we illuminate new areas for research into the logic of war expansion at this critical moment in history.

7 Discussion

We identify a trade-off between the amount that states can extract from peace when they negotiate world order, and the scope of war contagion should world order collapse. We use this trade-off to illuminate several important debates in international relations. We explain that great powers tolerate a fragile peace in which a single crisis could cause world war because the benefits from peace today outweigh the cost of catastrophic war when the risk of war initiation is low.

We explain that multipolar systems can facilitate more efficient (or more extractive) agreements than bipolar systems because they can be supported by a wide range of interconnected threats. The diversity of potential threats provides one explanation for differences in pathways of war contagion that follow from seemingly similar agreements. Thus, it is plausible to find examples where contagion follows based on trade dynamics in some cases, but geographical features, alliance dynamics or power in others.

We show that extractive peace settlements—settlements that overwhelmingly benefit a single country—are a primary source of the worst forms of war contagion. Highly extractive, multi-polar agreements can lead rational, fully-informed states to initiate war. This calls into question whether we should venerate skilled diplomats, like Metternich, who achieve large settlements for their state. They not only exploit others in peace, they bear responsibility for the ways local conflicts expand into massive, continental wars.

In section 6 we illuminate aspects of these findings through two important and under-studied cases. In section 5 we detail the implications for theories of war contagion and diplomacy. We conclude with a policy implication.

In the coming decades China and Russia will call on the US to renegotiate world order. Many scholars tell policymakers that the US will be better off if we construct complicated institutional agreements that link issues together because these agreements let us make the most out of peace (Keohane 2005). Our theory suggests caution. By striking a complicated agreement to extract the most from peace, we increase the risk that any small crisis will unravel into catastrophic war. Each layer of complexity raises the risk that wars will involve multiple actors and issues if peace fails. Simpler agreements that leave the US worse off in peace will minimize contagious conflict when a crisis erupts.

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A Baseline proofs

Our analysis plan is as follows. Since we do not assume a bargaining protocol, we characterize all the equilibria in the model. We show that some of equilibria sustain peace with stable threats of war contagion. But these threats can only sustain a limited set of offers. We then search for the Pareto dominant equilibria. We show that these equilibria sustain peace. But that they require reciprocal threats of war expansion. The efficiency-stability trade-off follows.

A.1 Lemma 2.1: Stable equilibria

The war strategies that support Lemma 2.1 are $s^A = s^B = \omega_1 = 0|h(.), \omega_2 = 0|h(.)$. That is no player declares war for any history.

Consider a sub-game $\omega = 1$. A prefers to deviate to war over issue 2 iff: $\alpha_2 p_2 - w < q_2$. B prefers to deviate if: $\beta_2(1 - p_2) - w < 1 - q_2$. These solve for the bounds of $q_2^\dagger$ as desired. A similar argument derives the bounds on $q_1^\dagger$ following sub-game $\omega = 2$. It follows that no state can profit from war expansion. Since players cannot threaten war expansion, these inequality comparisons are also sufficient to show that no player can profit from deviation to war initiation.

A.1.1 Ranges of payoff values for stable equilibria

Later on we will show that there is a trade-off between stability and efficiency. That analysis requires us to understand Pareto-dominant equilibria.

The following facts about the stable equilibria and their payoffs will help. First, focusing only on equilibria described in Lemma 2.1, the following offer pairs cannot be Pareto-dominated by another equilibrium in which $q^\dagger$ is peacefully sustained:

- A's preferred set: $q_1 = p_1 + w/\beta_1, q_2 \in q_2^\dagger$
- B's preferred set: $q_1 \in q_1^\dagger, q_2 = 1 - p_1 - w/\alpha_2$

Notice, that there is one result that is common in both sets—$q_1 = p_1 + w/\beta_1, q_2 = 1 - p_1 - w/\alpha_2$. This result is special because it provides each state with the largest share of their favourite issue. This generates: $U^A = \alpha_1(p_1 + w/\beta_1) + \alpha_2(p_2 - w/\alpha_2), U^B = \beta_1(1 - p_1 - w/\beta_1) + \beta_2(1 - p_1 + w/\alpha_2)$

A.2 Contagious Equilibria

Like stable, contagious refers to the nature of threats that sustain a peaceful offer.

We now focus on a set of offers that we call **traded offers**. We define any offer in the following set of offers as traded offers:

$$\tilde{q} = \tilde{q}_1 > \max(q_1^\dagger), \tilde{q}_2 < \min(q_2^\dagger). \tag{1}$$

Notice that $q^\dagger \cap \tilde{q} = \emptyset$.

We define a reciprocal punishment strategy as a combination of war rules that include three features. First, neither state in a dyad initiates war against the other: $w^j = 0|\omega = \emptyset$. Second, at least one state commits to expanding the war to the other issue they contest given any decision to initiate war. Third, both states in a dyad commit to expanding the war under at least one condition (i.e. no player plays a no war strategy). That is: $w^A = 2|\omega = 1, w^B = 1|\omega = 2, w^A = w^B = 0|\omega = 0$. But it also includes, $w^A = 2|\omega = 1, w^B = 1|\omega = 2, w^B = 1|\omega = 1, w^A = w^B = 0|\omega = 0$.

Proposition A.1 *Consider an initial offer $q \in \tilde{q}$ that also satisfies:*

$$\beta_1(q_1 - p_1) + \beta_2(q_2 - p_2) < 2w < \alpha_1(q_1 - p_1) + \alpha_2(q_2 - p_2). \tag{2}$$

Then any reciprocal punishment strategy is an SPE.

Proof. Consider the sub-game after $\omega = 2$. On the path (at least) B expands the war to issue 1. B can threaten to do so if $q_1 > \beta_1(1 - p_1) - w$. This is equivelent to $q_1 > \max(q_1^\dagger)$ as desired.[32]

Consider the sub-game after $\omega = 1$. On the path (at least) A expands the war to issue 2. A can threaten to do so if $q_2 < \alpha_2 p_2 - w$. This is equivalent to $q_2 < \min(q_2^\dagger)$ as desired.

Working backwards, we claim that no states wants to initiate war. On the path, any local war triggers total war. This yields $U^A(\omega = 1, 2) = p_1\alpha_1 + p_2\alpha_2 - 2w, U^B(\omega = 1, 2) = (1 - p_1)\beta_1 + (1 - p_2)\beta_2 - 2w$. If instead, there is no war players utility from accepting the offer on the path is: $U^A(\omega = 0) = \alpha_1, U^B(\omega = 0) = \beta_2$. From contrasting these inequalities, we derive Condition 2. This completes the proof.

We also assume that each kind of equilibria employs a distinct punishment strategy. In particular, we showed that the efficient equilibria could be supported by reciprical (and contagious) punishment. We'll now show that this focus is warranted:

Lemma A.2 *Assume condition 2 is satisfied. We cannot peacefully sustain $q \in \tilde{q}$ without contagious threat of war.*

First, consider a punishment strategy in which following a war over issue 2, A punishes reciprocally with probability γ_A and does punish with probability $1 - \gamma_A$. Now consider the sub-game after war over issue 2 and which A has decided to punish independently. By construction of $\tilde{q}$, A can profitably deviate to punishment. A similar argument is made for B's punishment following a war over issue 1. This completes the proof.

A.3 Proof of proposition 2.2

Proposition 2.2 claimed that there is an efficiency stability trade-off. We prove it in two steps. First, we characterize the Pareto dominant equilibria. Second, we use the results above to point out that at least one offer in $\tilde{q}$ dominate $q^\dagger$.

Define a fixed length of q as $t = q_1 + q_2 + \epsilon - \epsilon$.

Lemma A.3 *Consider all of the offers that hold t constant. Of these offers, the Pareto dominant equilibria sustain peace and maximize q_1 or minimize q_2.*

Holding constant the length of an offer t we can re-write player utilities from peace as: $U^A\alpha_1(q_1 + \epsilon) + \alpha_2(q_2 - \epsilon), U^B\beta_1(1 - q_1 - \epsilon) + \beta_2(1 - q_2 + \epsilon)$. Notice, that both U^A, U^B are increasing in ϵ.

In section A.1 we showed that all of the non-contagious equilibria were bounded by $q \in q^\dagger$. In section A.2 we showed that contagious equilibria exist. By definition 1, $\tilde{q}_1 > q_1^\dagger, \tilde{q}_2 < q_2^\dagger$. This completes the proof.

A.4 The equilibria in Lemma 2.3 and 2.4 are Pareto-dominant

The equilibria that we highlight in the manuscript are examples of those described in proposition A.1. Inequality 2 simply re-states 1. Condition 2.3 assumes condition 2 is satisfied with the additional restriction $q_1 = 1, q_2 = 0$. The proof of proposition A.1 is sufficient to prove the two equilibria stated in the manuscript.

[32] In the grimm trigger variants of reciprocal punishment, A cannot profitably deviate to no war because B will expand to war anyway. We include these for completeness.

We claim that the equilibria in Lemmas 2.3 and 2.4 are Pareto efficient give $\mathcal{A}_1$ and condition 2.3. We now explain why this is the case. Starting with the welfare maximizing equilibrium. Notice that if $\mathcal{A}_1$ holds, this equilibrium generates the maximum total welfare that the game allows for $U^A + U^B = \alpha_1 + \beta_2$. Equilibria must be Pareto dominant if they generate the maximum total welfare.

Turning to the extractive equilibrium. We claim that the equilibrium generates that largest total expected utility for A. Any equilibrium that leaves A with her largest total expected utility must be Pareto dominant.

For the equilibrium to hold together, A must be able to threaten war over issue 2 if B fights for issue 1. This means that $\alpha_2 q_2 > \alpha_2 p_2 - w$. This is the first minimum condition on q_2. It also must be that B prefers to accept the offer rather than fight a total war. This is true if: $\beta_2(1-q_2) > \beta_1(1-p_1) + \beta_2(1-p_2) - w$. This solves for the second minimum bound on q_2.

By construction, the equilibrium describes the offer that leaves A with all of A's favourite issue and the largest amount of A's least favourite issue for which we can support reciprocal punishment in an SPE.

What's left to show is that there is no SPE in which A does better that includes war on the path. Clearly, A cannot do better from total war because that deviation is allowable and A does not take it. Now consider an equilibrium in which players fight a war over issue 1. We've shown that in this sub-game we can only sustain peace over issue 2 if $q_2 \in q^\dagger$. We've also shown that A does better from the extractive equilibrium than from $p_1\alpha_1 - w + \alpha_2(p_2 + w/\beta_2)$. A similar argument can be made in the sub-game that follows a war over issue 2. Thus, A cannot secure a larger utility in any equilibrium that includes war on the path.

Lemma A.4 *A "maximally-extractive" equilibrium maximizes B's utility. In this equilibrium,* $q_1 = \max(p_1 + \frac{w}{\beta_1}, \frac{\alpha_1 p_1 + \alpha_2 p_2 - 2w}{\alpha_1})$ *and* $q_2 = 0$. *This settlement can only be peacefully sustained with a reciprocal threat of war expansion.*

The equilibrium is constructed in the same was as Lemma 2.4 but we seek out the offers that maximize B's utility.

A.5 Ruling out other offers

So far, we have shown that a punishment strategy exists that supports peace for offers in $q^\dagger$ and also in $\tilde{q}$ if $\mathcal{C}_1$ is satisfied and not otherwise. There is one other class of offers. We now show that we cannot peacefully sustain them with any punishment strategy. This will make it easier to locate pareto dominant equilibria.

Lemma A.5 *We cannot sustain peace in an SPE in which* $q_1 \in q_1^\dagger, q_2 \notin q_2^\dagger$.

Arguing by contradiction, assume that we can support this offer with some punishment strategies that ensures no player initiates war. Then show that at least one player can profitable deviate to war. Consider the subgame that follows after war over issue 2. By Lemma 2.1, no player can credibly threaten to fight a war over issue 1. It follows that the equilibrium must include a punishment strategy in which if players fight over issue 2, then no player expands the war to issue 1.

In one case we set $p_2 - w/\alpha_2 > q_2 \notin q_2^\dagger$. For this equilibrium to sustain peace, A cannot initiate war with certainty over any issue. We've shown that if A does initiate war over issue 2, that the war cannot spread to issue 1 in any SPE. Thus, A cannot profitably deviate to war over issue 2 if: $\alpha_1 q_1 + \alpha_2 q_2 > \alpha_1 q_1 + \alpha_2 p_2 - w \implies p_2 - w/\alpha_2 < q_2$, a contradiction. A similar argument can be made by considering a value of q_2 that B will not accept. By symmetry, an identical argument can be made for $q_1 \notin q_1^\dagger, q_2 \in q_2^\dagger$.

We also cannot sustain peace for an offer in which $q_i > \max(q_i^\dagger)$ or $q_i < \min(q_i^\dagger)$ for $i \in 1, 2$. The reason is that at least one state can profitably deviate to total war.

A.6 Classifying all of the Pareto Dominant equilibria

Summing up, we've identified the boundaries of offers in $\tilde{q}$ that can sustain peace. These boundaries maximized $q_1 = 1$. We've also showed that for any fixed t, offers that sustain peace and maximize q_1 dominate all others of the same length.

The last thing to show is that each of these are pareto-dominant.

Lemma A.6 *If $\mathcal{C}_1$ is satisfied, then there are two sets of offers that, with the threat of reciprocal punishment are pareto optimal equilibria. One set of these offers benefits A: $q_1 = 1, q_2 \in \tilde{q}_2, q_2 < \min(p_2 - \frac{w}{\alpha_2}, \frac{2w-\beta_1(1-p_1)+\beta_2 p_2}{\beta_2})$. The other set of offers benefits B: $q_1 =\in \tilde{q}_1, q_1 > \max(p_1 + \frac{w}{\beta_1}, \frac{\alpha_1 p_1 + \alpha_2 p_2 - 2w}{\alpha_1}), q_2 = 0$*

Notice that these sets only share one offer $q_1 = 1, q_2 = 0$. Furthermore, the bounds on these offers ensure that we can peacefully sustain them given what we have already shown.

Notice that each set contains offers of different lengths t, and each specific offer in the set that benefits A maximizes q_1, and each specific offer in set that benefits B minimizes q_2. We've already shown that offers that maximize q_1 or minimize q_2 dominate all other offers of the same length. Since each of these offers pareto dominate all others of the same length, we need only contrast the conjectured offers against each other.

Consider an offer in the set that benefits A with a specific $q_2 = q_2^x$ as a parameter. A does worse with any offer that is also in that set, and includes $q_2^y < q_2^x$. B does worse with an any offer that is also in that set, and includes $q_2^y > q_2^x$. Finally, notice that the offer in the set that benefits A that leaves A with the smallest amount of utility $q_1 = 1, q_2 = 0$ is identical to the offer in set B that leaves A with the largest amount of utility. It follows, that switching from an offer in the set that benefits A to the set that benefits B must make A worse off.

We can apply the same argument to set B. This completes the proof.

B Re-negotiation and the risk of an accident

We've shown that the renegotiated settlement q^r and punishment strategy of independent (stable) conflict is sub-game perfect. It follows that the risk of a local war over issue 1 is $\psi_1(1-\psi_2)$ (costing w), and the risk of a global war is $\psi_1\psi_2$ (costing $2w$). A's utility from renegotiation to q^r given the risk of an accident is: $U^A(q^r) = p_1\alpha_1 + p_2\alpha_2 - w(\psi_1 + \psi_2)$. B's is: $U^B(q^r) = (1-p_1)\beta_1 + (1-p_2)\beta_2 - w(\psi_1 + \psi_2)$

We've also characterized the efficient equilibria, which are contagious. It follows that the risk of a global war is $1 - (1-\psi_1)(1-\psi_2) = \psi_1 + \psi_2 - \psi_1\psi_2$ and the risk of a local war is 0. A's utility from retaining the initial settlement given the risk of an accident is: $U^A(q) = (1 - \psi_1 - \psi_2 + \psi_1\psi_2)(q_1\alpha_1 + q_{22}) + (\psi_1 + \psi_2 - \psi_1\psi_2)(p_1\alpha_1 + p_2\alpha_2 - 2w)$.

B's utility is: $U^B(q) = (1 - \psi_1 - \psi_2 + \psi_1\psi_2)((1-q_1)\beta_1 + (1-q_2)\beta_2) + (\psi_1 + \psi_2 - \psi_1\psi_2)((1 - p_1)\beta_1 + (1-p_2)\beta_2 - 2w)$.

Players' decisions to vote for re-negotiation is a simple utility comparison. A votes to re-negotiate if:

$$\frac{\psi_1 + \psi_2 - 2\psi_1\psi_2}{1 - \psi_1 - \psi_2 + \psi_1\psi_2} > \frac{\alpha_1(q_1 - p_1) - \alpha_2(p_2 - q_2)}{w}$$

Otherwise, A does not re-negotiate. Recall that in any efficient settlement $q_1 > p_1, p_2 > q_2$.

B votes to renegotiate if:

$$\frac{\psi_1 + \psi_2 - 2\psi_1\psi_2}{1 - \psi_1 - \psi_2 + \psi_1\psi_2} > \frac{\beta_2(p_2 - q_2) - \beta_1(q_1 - p_1)}{w}$$

Otherwise, B does not re-negotiate.

These conditions are jointly satisfied as $\psi_1, \psi_2 \to 1.$ or if $w \to \infty$. Thus, these parameters determine whether both or neither players want to renegotiate.

Without loss of generality, we focus on the case where A does not want to renegotiate but B does. This happens if:

$$\alpha_1(q_1 - p_1) - \alpha_2(p_2 - q_2) > w\frac{\psi_1 + \psi_2 - 2\psi_1\psi_2}{1 - \psi_1 - \psi_2 + \psi_1\psi_2} > \beta_2(p_2 - q_2) - \beta_1(q_1 - p_1) \tag{3}$$

The Implication derived in the manuscript, is taken from partial derivatives for $(\alpha_1 + \beta_1)(q_1 - p_1) - (\alpha_2 + \beta_2)(p_2 - q_2)$.

B.1 Re-negotiating over one territory while fighting over another.

In this section we consider the possibility that states can renegotiate over one set of territories as they fight a war over a different set of territory.

We start with the assumption of an initial settlement, $\tilde{q} = q_1 = 1, q_2 = 0$.

We define a re-negotiation settlement once war has started: $q_1^r = p_1, q_2^r = p_2$.

We now examine the following model.

1. Players simultaneously decide to accept peace over all issues or declare war over one or both issues they contest.
2. If all states accept peace, the game ends in peace.
3. If both wars are declared, the game ends in total war.
4. If one war is declared, then any player that had war declared against them decides between:
   (a) Accepting peace. In which case the game ends in a local war and a remaining settlement associated with $\tilde{q}$
   (b) Demanding the re-negotiated settlement. In which case the game ends in a local war and a settlement associated with q^r.
   (c) Expanding the war. In which case the game ends in total war.

The possibility for war-time negotiation poses an important concern. Reciprocal threats allow for efficient offers because they make states vulnerable. That is, A's threat to fight over issue 2 is credible only because A gets less than her minimum demand from fighting. However, if we allow for war-time renegotiation, A's threat of war is not credible once war has started. Therefore, we might expect all of the efficient equilibria to unravel leading to war given the settlement $\tilde{q}$.

Lemma B.1 *In the model that allows for war-time renegotiation, if $\mathcal{C}_1$ is satisfied and $\alpha_1 > \frac{\alpha_2 p_2 - w}{1 - p_1}$, $\beta_2 > \frac{\beta_1(1-p_1)-w}{p_2}$ then there is an equilibrium that sustains global peace. Off the path, if B declares war over issue 1, A selects q_2^r. If A declares war over issue 2, A selects q_1^r.*

The logic behind this result is that efficient settlements are efficient because states get their preferred settlement. Both war and the reneogtiated settlements are efficient because issues are split (in expectation) proportionate to power. Realizing this, states never initiate war. From A's perspective

A prefers global peace to initiating a war knowing it will cause B to re-negotiate iff: $\alpha_1 > \alpha_1 p_1 + \alpha_2 p_2 - w$. B prefers global peace to initiating a war knowing it will cause A to re-negotiate iff: $\beta_1 > \beta_1(1 - p_1) + \beta_2(1 - p_2) - w$. These solve for the conditions as stated in the equilibrium.

The model above assumed that war-time renegotiation is costless. However, we now introduce a penalty that both states pay if either chooses to re-negotiate over one issue as they fight over another $r \geq 0$. We introduce this cost for substantive reasons. Specifically, when states are fighting a brutal war with a rival in one theatre, it is very hard to explain to their selectorate why they would make territorial concessions somewhere else and not simultaneously resolve the underlying conflict. This is not to say that states do not link issues in compromise once a war has concluded. However, it is a different issue for states to redistribute peaceful territory as they continue to battle each other somewhere else. We could not find a single case in history in which states considered making an offer like this.

As a result of this re-negotiation cost, if states

Lemma B.2 *In the model that allows for costly war-time renegotiation, we can sustain peace under two condition. If $R \geq w$, then reciprocal threats of war sustain peace as described in lemma 2.3. If $R \leq w$ and $\alpha_1 > \frac{\alpha_2 p_2 - w - R}{1 - p_1}$, $\beta_2 > \frac{\beta_1(1-p_1) - w - R}{p_2}$ then global peace is sustained backed by the threat of costly re-negotiation as described in Lemma B.1.*

$R = w$ is the point that a player is indifferent between a local war or re-negotiation.

C Multipolarity

C.1 Proof of Propositions 4.1-4.2

Propositions 4.1 and 4.2 only assert existence. We can establish both with a single example. Here, we complete the outline of the example offered in the paper. From the table, we have an initial offer $\tilde{q} = [\tilde{q}_{AB}, \tilde{q}_{BA}, \tilde{q}_{BC}, \tilde{q}_{CB}, \tilde{q}_{CA}, \tilde{q}_{AC}] = (1, 0, .98\bar{3}, \frac{11}{36}, 1, \frac{5}{6})$. We also have parameters $p_{CA} = \frac{1}{12}$, $p_{CB} = \frac{1}{6}$, $p_{AB} = \frac{3}{4}$, $L = \frac{3}{5}$, $w = \frac{1}{12}$. Condition $\mathcal{C}_2$ allows us to derive the remaining parameters.

The following strategies are an equilibrium:

A: Accept $\tilde{q}$. If $(\omega \cap \{AB, BA\}) \neq \emptyset$, attack CA; otherwise, do not attack any issue.

B: Accept $\tilde{q}$. If $\omega \ni BC$, attack BA and AB; otherwise, do not attack any issue.

C: Accept $\tilde{q}$. If $\omega \ni CA$, attack BC; otherwise, do not attack any issue.

First, note that $\tilde{q}_{CB}$ and $\tilde{q}_{AC}$ cannot be contested by a player unless the player already expects the other to attack that issue. The issues are *closed*, i.e. no player can credibly threaten to attack them. In the equilibrium, since no player ever attacks CB or AC, then no player can profitably or plausibly deviate and attack either CB or AC in any SPE. We can therefore rule them out. Moreover, note that subgame perfection requires that B cannot attack BC nor A AB,BA for the same reasons.

We can now easily verify that no player desires to deviate overall:

$$EU_A[\text{accept } \tilde{q}] = \frac{11}{6} + L = 2.4\bar{3}$$
$$EU_A[\text{attack } CA] = p_{AC}L - w + p_{AB}(1+L) - 2w + \tilde{q}_{AC} = \frac{11}{12} + \frac{3}{4}\frac{8}{5} - \frac{3}{12} = 2.\bar{3}$$

$$EU_B[\text{accept } \tilde{q}] = .98\bar{3} + \frac{25}{36}\frac{3}{5} = 1.4$$
$$EU_B[\text{attack } AB] = EU_B[\text{attack } BA] = p_{BA}(1+L) - 2w + p_{BC} - w + (1-\tilde{q}_{CB})L = 1.4$$

$$EU_C[\text{accept } \tilde{q}] \approx 1.42$$
$$EU_C[\text{attack } BC] = pCAL - w + p_{CB}L - w + \tilde{q}_{CB} + (1 - \tilde{q}_{AC}L = .08\bar{3}$$

We can also easily verify that players' threats satisfy subgame perfection. Below are the relevant utility comparisons.

$$EU_A[\text{attack } CA;\ (\omega \cap \{AB, BA\}) \neq \emptyset] = 2.\bar{3}$$
$$EU_A[\text{do not attack } CA;\ \omega \ni AB \text{ and } \omega \not\ni BA] = p_{AB} - w + \tilde{q}_{AC} + L(2 - \tilde{q}_{BA} - \tilde{q}_{CA}) = 2.1$$
$$EU_A[\text{do not attack } CA;\ \omega \ni BA \text{ and } \omega \not\ni AB] = 2.2$$

$$EU_B[\text{attack } AB \text{ and } BA;\ \omega \ni BC] = 1.4$$
$$EU_B[\text{do not attack } AB \text{ or } BA;\ \omega \ni BC] = 1.1\bar{6}$$
$$EU_B[\text{only attack } BA;\ \omega \ni BC] = 1.\bar{3}$$

$$EU_C[\text{attack } BC;\ \omega \ni CA] = .08\bar{3}$$
$$EU_C[\text{do not attack } CA;\ \omega \ni CA] = .41\bar{5}$$

C.1.1 Generalization of results

We now prove two results that make explicit complex threats are necessary to generate offers described above.

Proposition C.1 *We cannot sustain peace agreements that have the features described in proposition 4.1-4.2 with exclusively reciprocal threats of war.*

Proposition C.2 *Under $\mathcal{C}_1$ and $\mathcal{C}_2$, if a welfare-maximizing settlement would be impossible under bipolarity, then it is also impossible under multipolarity. If a welfare-maximizing settlement would be possible under bipolarity, then under conditions $\mathcal{C}_1$ and $\mathcal{C}_2$, it must be sustained by reciprocal threats in multipolarity.*

Proposition C.1 follows obviously from the bipolar proofs and from C.1 above.

Proposition C.2 depends on condition $\mathcal{C}_2$. (It is straightforward to think of counterexamples when the condition is violated.)

Proof Define $\phi \equiv \frac{1+2w}{1+L}$. By Lemma 2.3, a welfare-maximizing equilibrium does not exist under bipolarity between i and j whenever $p_{ij} > \phi$. If $p_{AC} \leq \phi$, then $\mathcal{C}_2$ guarantees $p_{ij} \leq \phi$ for all i, j, and a welfare-maximizing equilibrium must exist by Lemma 2.3. If $p_{AB} > \phi$, then $p_{AC} > \phi$. It will therefore suffice to prove that, if $p_{AC} > \phi$, then a welfare-maximizing equilibrium does not exist.

In a welfare-maximizing equilibrium, $q_{ij} = 1$ for all i, j. Consider the A-C dyad. A must decide whether to attack CA. If C never attacks BC in retaliation when C attacks CA, then A is strictly better off attacking CA, since $p_{AC} > \phi$. Therefore, it must be that C would expand the war to the C-B dyad with nonzero probability, should A attack CA. Since C cannot threaten CB, it must attack BC. Now consider player B's decision in a subgame in which A has attacked CA and C has attacked BC. There are two possibilities: i) A will not attack either AB or BA this round; or ii) A will attack AB or (inclusive) BA this round.

Consider (i). In this case, B must decide whether to expand the war to the A-B dyad or not. B is strictly better off retaliating against CB, not expanding the war, since $p_{BA}(1 + L) - 2w < 1 = q_{BA}$ and $p_{BC}L - w > 0$. Knowing this, in the original subgame where A has attacked CA, C's utility to expanding the war is strictly negative, since B will certainly retaliate against CB, and $q_{CB} = 1 > p_{CB}(1 + L) - 2w$. C therefore cannot credibly threaten to expand the war should A attack CA, and so a welfare-maximizing equilibrium is impossible.

Consider (ii). In this case, B expects to fight a war over at least one issue in the A-B dyad because A will initiate war. Necessarily, this means C must have attacked AC, since otherwise A would not be given the opportunity to attack B. If B expects war over BA, then B is strictly better off attacking both AB and CB, but this eliminates C's credible escalatory threat, as in (i) above. If this issue is AB, then B will not attack BA (though A will certainly do so in the next round), but it will attack CB. As in (i), this again denies C its escalatory threat against A, and so a welfare-maximizing equilibrium is impossible.

A similar logic will establish that if $q_{ij} = 1$ for all issues, and the settlement is peaceful, then $\mathcal{C}_1$ and $\mathcal{C}_2$ guarantee that peace is sustained by reciprocal threats.

C.2 Volatility Examples

C.2.1 Example: Modest Volatility

Consider the following example in which A is the strongest player, but not so strong that $p_{AC} \geq \frac{1+2w}{1+L}$. (In other words, a welfare-maximizing settlement is feasible.) Let $p_{AC} = \frac{2}{3}$, $p_{BC} = .6$, $p_{AB} = .6$, with $p_{CA} = 1 - p_{AC}$ and similarly. Note that p satisfies $\mathcal{C}_1$ and $\mathcal{C}_2$. Consider this settlement:

$$\begin{aligned}
&\text{A gets: } q_{AC} = q_{AB} = 1,\ q_{BA} = p_{BA} - w \\
&\text{B gets: } q_{BC} = 1,\ q_{BA} = p_{BA} - w,\ q_{CB} = p_{BC} + w/L \\
&\text{C gets: } q_{CA} = 1,\ q_{CB} = p_{BC} + w/L
\end{aligned}$$

This settlement can form part of a volatile equilibrium in which C attacks AC with probability $x_C \approx .42$ and B attacks BA with probability $x_B \approx .28$. A only retaliates when *both* players attack. The equilibrium strategies are as follows:

A: accept q. Attack CA iff ω includes two of AB, AC.

B: accept q with probability $(1 - x_B)$ and attack AB with probability $x_B \approx .28$, where x_B makes C indifferent between attacking AC and not attacking. If $\omega \ni BC$, attack AB.

C: accept q with probability $(1 - x_C)$ and attack AC with probability $x_C \approx .42$, where x_C makes B indifferent between attacking AB and not attacking. If $\omega \ni CA$, attack AC and BC.

Below are the relevant utility comparisons:

$EU_A[\text{accept } q] \approx 2.31$
$EU_A[\text{attack } CA] = (p_{AC} + p_{AB})(1 + L) - 3w = 1.52$
$EU_A[\neg\text{attack } CA; \omega \ni AB,\ \omega \not\ni AC] \approx 2.01$
$EU_A[\neg\text{attack } CA; \omega \ni AC,\ \omega \not\ni AB] \approx 1.94$

$EU_B[\text{accept q}] = 1 + q_{CB}L = 1.54$
$EU_B[\text{attack AB}] = (1 - x_C)(1 + q_{CB} + p_{BA}L - w) + x_c(p_{BA}L + p_{CB} - 2w) + (1 - q_{CB})L + q_{BA} = 1.54$

$EU_C[\text{accept q}] = 1 + q_{CB} = 1.68$
$EU_C[\text{attack } BC] = p_{CA}(1 + L) + p_{CB}L - w \approx 1.31$

Why would players choose such a world? In the paper, we address two rational reasons. Here, it is worth considering another: constrained proposal power. Suppose $q_{CA} = 1$ has already been fixed by player C, but A has proposal power over all other issues. Then following are the best A can do from its possible proposals:

	Volatility	**Reciprocity**	**Simple Chain Rxn**	**Stability with B**
A's Exp. Util. $\lvert q_{CA} = 1$	2.31	2.276	2.14	1.87

In essence, C has tried to put A in a situation where A must choose between simple reciprocity (in which case C will get a welfare-maximizing settlement) or a "hired thug" chain reaction (in which C can hide behind B's superior firepower). A is able to escape this dilemma by closing BA, thus allowing it to extract *all* of AB and AC, while putting B and C in a position where each will check the other. Given the constraints on A's proposal power, The volatile settlement is optimal. We address this option a bit more in section C.2.3 below.

C.2.2 Example: Extreme Volatility

Consider the following example in which A is dominant. We will construct an example so that A extracts *all* of AC, AB, and BA. Let $p_{AC} = \frac{7}{8}$, $p_{AB} = \frac{7}{10}$, $p_{BC} = \frac{6}{10}$, with $p_{CA} = 1 - p_{AC}$ and similarly. Note that p satisfies $\mathcal{C}_1$ and $\mathcal{C}_2$. Let $w = \frac{1}{20}$ and $L = \frac{6}{10}$. Consider this settlement:

A gets: $q_{AC} = q_{AB} = 1 - q_{BA} = 1$
B gets: $q_{BC} = 1,\ q_{CB} = p_{BC}L + w/L$
C gets: $q_{CA} = 1,\ q_{CB} = p_{BC}L + w/L$

This settlement can form part of a volatile equilibrium in which C attacks AC with probability $x_C \approx .78$ and B attacks BA with probability $x_B \approx .03$. A only retaliates when *both* players attack. The equilibrium strategies are as follows:

A: accept q. Attack CA iff ω includes two of AC, AB, BA.

B: accept q with probability $(1 - x_B)$ and attack BA with probability $x_B \approx .034$. If $\omega \ni AB$ or BC, attack AB and BA.

C: accept q with probability $(1 - x_C)$ and attack AC with probability $x_C \approx .78$ If $\omega \ni CA$, attack AC and BC.

Below are the relevant utility comparisons:

$$EU_A[\text{accept } q] \approx 2.46$$
$$EU_A[\text{attack } CA] = (p_{AC} + p_{AB})(1 + L) - 4w = 2.32$$
$$EU_A[\neg\text{attack } CA; \omega \ni BA,\ \omega \not\ni AC, AB] = 2 + p_{AB}L - w = 2.37$$
$$EU_A[\neg\text{attack } CA; \omega \ni AC,\ \omega \not\ni AB, BA] = 1 + L + p_{AC} - w \approx 2.42$$

$$EU_B[\text{accept q}] = 1 + q_{CB}L = 1.44$$
$$EU_B[\text{attack BA}] = (1 - x_C)(1 + q_{CB} + p_{BA} - w) + x_c(p_{BA}(1 + L) + p_{CB} - 3w) = 1.44$$
$$EU_B[\text{attack } BA \text{ and } AB] = p_{BA}(1 + L) + p_{CB} - 3w = 1.16$$

$$EU_C[\text{accept q}] = 1 + q_{CB} = 1.31$$
$$EU_C[\text{attack } BC] = p_{CA}(1 + L) + p_{CB}L - w = .29$$

A few things to note:

- Since A extracts all of AC, BA, and AB, A *cannot* credibly threaten to retaliate if only one of those issues is attacked.
- A is not at all frightened by C, but it can use the possibility C might attack AC to keep B in check, who is more powerful than C.

We now ask, why would the players ever desire to construct this system? In the paper, we gave two possible reasons. Above, we briefly mentioned a third. We now use the above example to demonstrate the logic of all three.

C.2.3 Constrained Proposal Power

Suppose $q_{CA} = 1$ and $q_{CB} = p_{BC}L - w$ are set exogenously. It doesn't matter why, especially; perhaps C enjoys some sort of first-mover advantage, such that it can "lock in" its favorite issues CA and CB to whichever values it chooses. It is easy to tell a story why C might try to lock in these values. These divisions might reflect military conquests on the ground. Or, C might be trying to create a "hired thug" scenario, as in table 3, where it can exploit B's superior power to improve its bargaining position vis-a-vis A. Whatever the reason, suppose A can then propose any settlement it wishes, subject to this constraint.

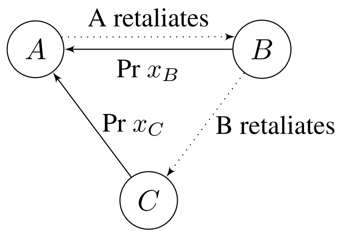
The best peaceful settlement A can propose gets it a utility of $\leq$ 2.45, less than its volatile payoff. Because A lacks a credible threat against *both* AB and BA, it must close one of these issues. It is best off closing $q_{BA} = p_{BA} - w$ and leaving open $q_{AB} = 1$ and $q_{AC} = 1$. This system is sustained by the contagion pathway (AB or AC) $\to CA \to BC$.

As a final option, A might simply choose to fight C and bargain bilaterally with B. This option yields ≤ 2.67, which is higher than its volatile payoff, assuming there is no risk of accident. If there is, volatility once again can dominate—see section C.2.4 below

C.2.4 Absorbs Exogenous Risks

The most fascinating feature of volatility is its ability to absorb risk.

First, consider what A could get purely from reciprocal settlements, i.e. tying AB only to BA, and AC only to CA. In this case, A would receive a utility of 2.845. Now suppose that there is some enormous



C attacks AC with probability x_C
B attacks AB with probability x_B
A attacks BA iff AC and AB have been attacked

B attacks CB only if A attacks BA

Figure 1: The logic of Volatility

exogenous risk of war $\psi_{AC} = .6$; in this case, reciprocity would get A less than 1.92. Yet, A's utility under volatility would remain *exactly the same*.

If A is constrained as in section C.2.3 above, then A might be tempted simply to fight C over AC and CA while negotiating reciprocally with B. If there is no risk of acident, this option yields ≤ 2.67. But even if there is a modest $\psi_{BA} = .05$, then A's expected utlity collapses to 2.32. Under these constraints, volatility becomes A's best option.

C.3 Proof of Proposition 4.3

Above, we offered examples of modest volatility and extreme volatility. Below, the proof will invoke general forms of these equilibrium patterns. For the proof, though, we will switch the circuit of the chain-reactive threat, so that (two of AC, CA, AB) $\rightarrow BA \rightarrow CB$. Modest volatility is visualized above in figure 1.

An issue ij is *closed* when $q_{ij} \in (\min\{p_{ij} - w, p_{ji} + w/L\}, \max\{p_{ij} - w, p_{ji} + w/L\})$. Define $\phi \equiv \frac{1+2w}{1+L}$.

Call a *modest V-settlement* a volatile equilibrium in which $q_{AC} = q_{AB} = q_{BA} = q_{CB} = 1$, and so are open under $\mathcal{C}_1$, while q_{CA} and q_{BC} are closed.

Lemma C.3 *Under the conditions in the proposition, if $p_{AB} < \phi$, then a modest V-settlement exists.*

Proof By construction. The following strategies must be an equilibrium under the conditions:

A: accept q. Attack BA iff ω includes both AB and AC.

B: accept q with probability $(1 - x_B)$ and attack AB with probability x_B s.t. $1 = (1 - x_B)(p_{CA}L - w) + x_B(p_{CA}L + p_{CB} - 2w)$. If $\omega \ni BA$, attack AB and CB.

C: accept q with probability $(1 - x_C)$ and attack CA with probability x_C s.t. $1 = (1 - x_C)(q_{BA} + p_{BA}L - w) + x_C(p_{BA}(1 + L) + p_{BC}L - 3w)$. If $\omega \ni CA$, attack AC.

This can be quickly verified. Note that the following comparisons omit "closed" issues because they do not affect marginal calculations.

Obviously, if $\omega = AC$, then A will not attack BA, since $p_{AB} < \phi$. If $\omega = AB$, then A's marginal calculation is

$$EU_A[\neg\text{attack } BA; \omega = AB] = 1 > p_{AB}L + p_{AC} - 2w = EU_A[\text{reject } q]$$

because

$$1 + 2w \geq p_{AC} + w/L + 2w > p_{AC} + w\frac{2L+1}{L} > p_{AC} + p_{AB}L$$

It follows obviously that A will not attack BA if $\omega = \emptyset$.

From B's perspective, its marginal calculation is

$$EU_B[\text{accept } q] = 1$$
$$EU_B[\text{attack } CB] = p_{BC}L + p_{BA}(1+L) - 3w < p_{BC}L + p_{BA} < 1$$

because $w > \frac{1}{3}p_{BA}L$ and $p_{BC} \leq p_{AB}$.

C's calculations follow obviously from B's, above. This completes the proof for the lemma.

Call an *extreme V-settlement* a volatile equilibrium in which $q_{AC} = q_{AB} = q_{BA} = 1 - q_{CA} = q_{CB} = 1$, and so are open under $\mathcal{C}_1$, while q_{BC} is closed.

Lemma C.4 *Under the conditions in the proposition, if $p_{AB} \geq \phi$, then an extreme V-settlement exists.*

Proof By construction. The following strategies must be an equilibrium under the conditions:

A: accept q. Attack BA iff ω includes two of AB, AC, and CA.

B: accept q with probability $(1 - x_B)$ and attack BA with probability x_B s.t. $1 = (1 - x_B)(p_{CA} + 1 - w) + x_B(p_{CA}(1+L) + p_{CB} - 2w)$. If $\omega \ni BA$, attack AB and CB.

C: accept q with probability $(1 - x_C)$ and attack CA with probability x_C s.t. $1 = (1 - x_C)(q_{BA} + p_{BA}L - w) + x_C(p_{BA}(1+L) + p_{BC}L - 3w)$. If $\omega \ni CB$, attack AC and CA.

This can be quickly verified. Obviously, if any two issues have been attacked, then A would attack BA, because $\phi \leq p_{AB}$. A's and B's calculations are essentially similar as in the proof for lemma C.3 above. It only remains to show that A will not retaliate if 0-1 issues have been attacked. This is obvious, because $L \leq .5$ and $p_{CA} > p_{AB}\frac{L}{1+2L}$. This completes the proof for the lemma.

The lemmas suffice to prove the proposition.